UNIVERSIDADE DE LISBOA
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Models and heuristics for forest management with environmental restrictions

" Documento Definitivo"

Doutoramento em Estatística e Investigação Operacional

Especialidade de Otimização

Teresa de Jesus Resende Silva dos Santos Neto

Tese orientada por:
Doutor Miguel Fragoso Constantino
Doutor João Pedro Pedroso
Doutora Isabel Martins

Documento especialmente elaborado para a obtenção do grau de doutor

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Resumo

A gestão florestal tornou-se, nas últimas décadas, bastante complexa. As primeiras abordagens à gestão florestal tiveram a madeira e a sua produção sustentável como objetivo principal. Desde que as preocupações ambientais se tornaram importantes para a sociedade, os mais recentes problemas de gestão florestal para produção de madeira incorporaram outros aspetos, tais como a proteção da vida selvagem, a manutenção da biodiversidade, o aumento da qualidade da água, a redução da erosão do solo e os aspetos estéticos.

O objetivo deste trabalho consistiu em desenvolver modelos matemáticos e implementar métodos de resolução em programação inteira para problemas de gestão florestal para produção da madeira com restrições ambientais. Muito trabalho foi desenvolvido nas últimas décadas nesta área, mas até à data, que eu conheça, ainda nenhum incluíu todas estas restrições. As florestas maduras concentram a maior parte da biodiversidade terrestre, nomeadamente, de espécies vegetais e animais e, por isso, foram as florestas consideradas neste trabalho.

Uma abordagem usual para incorporar as restrições ambientais tem sido a inclusão de restrições espaciais. Cada uma destas restrições dificulta substancialmente a resolução dos problemas especialmente quando as florestas são de grandes dimensões. Quatro tipos de restrições espaciais foram usadas, não todas ao mesmo tempo, definindo três problemas diferentes. As restrições usadas foram em: área de cada clareira, área total dos habitats, área interior total dos habitats e conetividade entre os habitats.

As restrições na área das clareiras limitam o tamanho das clareiras, diminuíndo assim os impactos ambientais provocados pelo corte das árvores. Contudo, não impedem uma dispersão das clareiras pela floresta e, consequentemente, uma floresta fragmentada.

A fragmentação de uma floresta consiste na divisão das manchas florestais em manchas mais pequenas, implicando uma redução na área total e na área interior das manchas e na conetividade entre as manchas. Estas manchas florestais podem tornar-se muito pequenas para serem habitats de muitas espécies, não só ao nível da área total, mas também da área interior, ou as distâncias entre os habitats podem tornar-se maiores do que aquelas que certas espécies podem percorrer. A área interior diminui em manchas mais pequenas ou com formatos mais alongados ou irregulares.

As políticas e regulações no corte das áreas florestais contemplando preocupações ambientais têm vindo a afirmar-se no contexto internacional. Foi realizada uma ligeira análise nas políticas e leis existentes em sete países em relação a quatro critérios. Austrália, Canadá,

Finlândia, Federação Russa, Suécia, Portugal e Estados Unidos da América foram os países selecionados em função da importância do setor florestal para o país ou no mundo, assim como a disponibilidade de informação. Considerando o contexto da tese e a informação disponível, os critérios analisados foram os seguintes: área máxima de corte, volume anual de corte, reflorestação e áreas protegidas.

Em relação aos limites impostos para a área de clareira, os valores diferem consideravelmente de um país para outro e, às vezes, no mesmo país, variam de acordo com as áreas geográficas, propriedade florestal (privado e público), tipos de floresta, métodos de corte ou outros critérios. Os limites variam entre 5-10 ha na Federação Russa e 260 ha em Ontario, no Canadá.

Volumes anuais de corte são requeridos pela maioria das jurisdições, calculados com base num rendimento sustentado, num fluxo uniforme ou numa variedade de fatores económicos, sociais e ambientais.

As políticas de reflorestamento que exigem níveis de estocagem ou prazos são encontradas em quase todos os países.

Todos os países analisados desenvolveram políticas ou leis nas áreas protegidas.

Portugal é o país com menos políticas em relação aos critérios referidos.

O trabalho foi estruturado em três artigos, onde em cada artigo foi estudado um dos problemas. Foram implementados dois métodos de resolução de pesquisa em árvore. Um dos métodos é de otimização multi-objetivo, pois o problema do último artigo é bi-objetivo. O tempo máximo de execução de cada método foi de duas horas, tendo estes funcionado como heurísticas para a maior parte das instâncias. Ambos os métodos usam uma árvore binária de pesquisa, a qual armazena uma sequência de decisões, em que cada decisão corresponde ao corte ou não de uma unidade de gestão num certo período.

Todos os problemas foram modelados com base na denominada formulação *cluster* para problemas de gestão florestal com restrições na área das clareiras.

Foram usadas 16 instâncias de teste, reais e hipotéticas, com um número de unidades de gestão variando entre 32 e 1363, e horizontes temporais variando entre 3 e 9 períodos.

O problema abordado no primeiro artigo é o da gestão de florestas para produção de madeira com restrições de volume, idade média final da floresta, área de cada clareira, área total dos habitats e conetividade entre os habitats. Por uma questão de simplicidade, considerou-se que um habitat é uma mancha madura com uma área total mínima (efeito de fronteira desprezado). As restrições de conetividade foram modeladas através de um índice, o qual satisfaz um conjunto de propriedades que qualquer índice de conetividade ideal deve verificar. Além disso, o seu valor é não crescente na árvore de pesquisa, sendo esta propriedade importante para o método de pesquisa em árvore implementado. Diferentes estratégias para guiar a procura na árvore foram usadas, assim como diferentes majorantes para parar a ramificação de um nodo da árvore. Quatro destes majorantes foram obtidos usando relaxações do problema de gestão florestal.

O segundo artigo substitui as restrições de conetividade pelas de área interior total dos habitats. Também foram consideradas restrições de volume, idade final média da floresta, área de cada clareira e área total dos habitats. Considerou-se que um habitat é uma mancha madura com uma área interior mínima (efeito de fronteira considerado). A estratégia para modelar a área interior não foi a mesma usada para a conetividade. Índices para medir a área interior consideram usualmente o formato das manchas florestais. Contudo, nem a área interior depende apenas da forma das manchas nem há nenhum índice que incorpore todas as características da forma das manchas. A área interior foi assim modelada diretamente usando o conceito de subregiões. Três diferentes tamanhos das zonas de fronteira foram usados. O método de pesquisa em árvore implementado no primeiro trabalho foi adaptado para resolver o problema.

No terceiro artigo foi abordado um problema bi-objetivo com restrições na área de cada clareira, na área total dos habitats e na área interior total dos habitats (também foram consideradas restrições de volume e idade final média da floresta). O valor atual líquido e as preocupações ambientais são objetivos usualmente de natureza conflituosa. Além disso, como a especificação de um valor restritivo para o índice de conetividade é frequentemente difícil, a maximização da conetividade entre os habitats foi adicionada à maximização do valor atual líquido. Um método de pesquisa em árvore baseado no método de Monte Carlo foi desenvolvido para encontrar as soluções eficientes do problema. Este método é usado como alternativa ao método *standard* de pesquisa em árvore, onde a construção e armazenamento da árvore é computacionalmente pesada, principalmente para as médias e grandes instâncias. Dado o seu carácter estocástico, para cada instância, o método foi executado doze vezes.

Os resultados mostraram que, nos dois primeiros artigos, o método de pesquisa em árvore encontrou boas soluções, algumas ótimas no primeiro estudo, num tempo razoável. Para a maioria das instâncias, a inclusão das restrições de área total de habitat e de conectividade (no primeiro artigo) ou de área interior total (no segundo artigo) implicou pequenas reduções no valor atual líquido obtido. A definição do que são florestas maduras pode ter proporcionado uma boa oferta de manchas maduras ao longo do tempo, e ser uma explicação para esta tendência.

Em todos os artigos, as restrições ambientais contribuíram de alguma forma para reduzir o efeito de fragmentação causado pela atividade de corte. As restrições ambientais consideradas em cada artigo ajudaram a aumentar a área total de habitat. Com o índice de conetividade, melhorou-se a conetividade entre os habitats no primeiro e terceiros artigos. O segundo artigo mostra que a dimensão das zonas de impacto influencia a disposição espacial dos habitats e, em geral, reduções na área interior estão relacionadas com maiores dimensões dessas zonas (maior efeito de fronteira). Em relação ao terceiro artigo, em cada execução do método foi obtido um número diferente de soluções eficientes devido à natureza estocástica do método. O número médio de soluções eficientes decresce significativamente quando são selecionadas as soluções eficientes obtidas em todas as execuções. Conclui-se assim que este método deve ser executado bastantes vezes para fornecer melhores aproximações da fronteira de Pareto.

Palavras chave: Planeamento florestal, Área interior, Conetividade, Programação Inteira, Programação multi-objectivo.

Abstract

The main focus of this thesis was to develop mathematical models and methods in integer programming for solving harvest scheduling problems with environmental restrictions. Constraints on maximum clearcut area, minimum total habitat area, minimum total core area and inter-habitat connectivity were addressed for this purpose.

The research was structured in a collection of three papers, each one describing the study of a different forest harvest scheduling problem with respect to the environmental constraints. Problems of papers 1 and 2 aim at maximizing the net present value. A bi-objective problem is considered in paper 3. The objectives are the maximization of the net present value and the maximization of the inter-habitat connectivity.

The tree search methods branch-and-bound and multiobjective Monte Carlo tree search were designed specifically to solve the problems. The methods could be used as heuristics, as a time limit of 2 hours was imposed. All harvest scheduling problems were based on the so-called cluster formulation. The proposed models and methods were tested with sixteen real and hypothetical instances ranging from small to large.

The results obtained for branch-and-bound and Monte Carlo tree search show that these methods were able to find solutions for all instances. The results suggest that it is possible to address the environmental restrictions with small reductions of the net present value. With respect to the forestry fragmentation caused by harvestings, the results suggest that, although clearcut size constraints tend to disperse clearcuts across the forest, compromising the development of large habitats, close to each other, the proposed models, with the other environmental constraints, attempt to mitigate this effect.

Keywords: Forest planning, Core area, Connectivity, Integer programming, Multiple objective programming.

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Contents

Chapter 1

Introduction

Forests play an important role in society, providing economic, social, cultural and environmental benefits. They are the source for many commercially products, ranging from timber and clothing to pharmaceuticals. At the same time, they also contribute with many services, such as clean air and water, wildlife habitat, carbon storage, regulation of the climate, beautiful scenery and places for recreation [McDermott et al., 2010].

In the past, forest management was applied principally to achieve the single objective of timber production. In the 70's a growing concern appeared particularly in native forests in relation to environmental concerns. The United Nation Conference on Environment and Development (UNCED), in Rio Janeiro 1992 [UNCED, 1992], has given forests an increasingly important role in the context of sustainable development and environmental conservation.

Many definitions have already been proposed for the concept of sustainable forest management. Even if there are differences between the definitions, most converges on the same aspects. In Europe, the concept was defined in 1993 at the pan-European Ministerial Conference as " The stewardship and use of forest lands in a way and at a rate that maintains their productivity, biodiversity, regeneration capacity, vitality and their potential to fulfill now and in the future relevant ecological, economic and social functions at local, national and global levels and that does not cause damage to other ecosystems".

Managing forests sustainably means using their benefits to meet society's needs in a way that conserves and maintains forest ecosystems for the benefit of present and future generations. It is not just about preserving the quantity of forests for future generations, it is also about respecting the biological diversity of the forests, the ecology of the species living within the forests, and the communities affected by the forests. Today, it is understood that environmental concerns, such as biodiversity, impact on climate, carbon cycle, water and soil are highly valued, even if not expressed in monetary terms.

Forest management has traditionally been a challenge for many researchers and practitioners of the Operational Research community (linear programming formulations of forestwide management planning problems were first introduced in the 1960s (*e.g.* Curtis [1962]).

Problems can range from harvesting, road building, transportation and preventing fire to production of pull-mills, paper-mills and heating plants. Many models and solution methods have been developed for such problems, in particular for the *harvest scheduling problem* (see Könnyű and Tóth [2013] for an overview).

The harvest scheduling problem can be solved in a strategic level, which involves long-term goals and is highly aggregated, in a tactical level, which focuses on medium-term or medium-scale goals and on the areas or trees to be harvested, or in an operational level, which states when and how the operations are performed. Strategic planning is generally nonspatial in nature, while tactical planning uses higher detail and often includes spatial considerations. Operational planning utilizes highly specific spatial information.

The harvest scheduling problem in its most basic form is to decide where and when to harvest, in order to maximize the net present value generated by the harvestings during a specific planning horizon. Typically, the forests are divided into stands and the planning horizons are discretized into periods. In general, a stand is an ecologically homogeneous forest area with respect to a selected set of properties, such as tree species composition, age of the trees and structure of the forest, and is managed in a similar manner.

Nearly all harvest scheduling problems have constraints on harvest levels. Typical requirements are that the forest produces a non-declining even flow of timber or a reasonable yield pattern, mainly to ensure that the industry is able to continue operating with similar levels of machine and labor utilizations. Other requirements aim to approximate the harvest levels to other commitments such as a maximum sustained volume at each period or increasing volumes over time. All these requirements can alternatively address product type.

Other common constraints require that at the end of the planning horizon the average age of the forest should be at least a certain target age, mainly to prevent harvestings in any part of the forest where any immediate profit can be made.

All these constraints are named non-spatial because the stand selection determined by them does not depend on the relative arrangement of stands.

For three decades now, the harvest scheduling problems have been addressing environmental concerns due to the impact of the harvesting activity on wildlife, biodiversity, soil, water, forest aesthetics, among others [Meneghin et al., 1988]. For this purpose, restrictions related to spatial characteristics of harvesting activity have been developed, mainly at the tactical and operational levels. The most common type of spatial constraints addressed is on the continuous harvested area, in which the extent of each clearcut is restricted. The reason for this measure is that when the area of each clearcut is sufficiently small, some impacts of harvesting activity are reduced, such as soil erosion, sediments in the water, deterioration of scenic beauty, among others. These constraints also often include adjacency requirements that restrict harvest in areas adjacent to clearcuts for specified time frames (as referred to as *greenup* time).

National forest programmes have become, almost universally, more environmentally oriented. New forest laws have been enacted or existing laws substantially amended since

UNCED to achieve the necessity of sustainable forest management. An analysis of some forest policies and laws that have emerged to address environmental concerns was performed in seven countries (Appendix A). Australia, Canada, Finland, Russian Federation, Sweden, Portugal and the United States of America were selected in function of the importance of the forest sector in each country or in the world, but fundamentally considering the availability of data and information. Since forest governance is largely handled at sub-national in Australia, Canada and the United States of America, five provinces/states were considered in each country: Quebec, Ontario, British Columbia, Alberta, New Brunswick provinces for Canada; Louisiana, Washington, Oregon, California, Alaska states for the United States of America; Queensland, New South Wales, Western Australia, Victoria, Tasmania states for Australia.

Given the significance of the environmental impacts of harvesting activity, many national policies and laws governing clearcutting set clearcut size limits. These limits differ considerably from one country to another and, sometimes, in the same country, vary according to geographic areas, forest ownership (private and public), forest types, harvest methods or other suitable classifications.

For example, in Australia, clearcut size limits range from 20 ha (regrowth karri forest type in Western Australia) to 120 ha (wet eucalypt per 5 years in Victoria). The limits on public lands in Canada range from 24 ha (spruce forest type in Alberta) to 260 ha (Ontario). In the United States of America, the limits on private lands range between 8.1 ha (California) and 48.5 ha (Oregon and Washington). Limits on national forests vary between 16.2 ha (forest types different from Douglas fir in Alaska, California, Oregon and Washington) and 40.5 ha (hemlock-Sitka spruce forest type in Alaska). The mandatory limits on public lands in Russia Federation vary from 5-10 ha (coniferous and broad-leaved of protection forests - group 1) to 250 ha (pioneer hardwoods in Far East). On private lands in Sweden, the limits are 20 ha (alpine forest type). There are no legal limits for clearcut sizes in Finland and Portugal.

Maximum clearcut size constraints eliminate large continuous harvested areas, but typically display a dispersion of smaller clearcuts across the forest and thus, a more fragmented forest [Franklin and Forman, 1987]. Forest fragmentation is the breaking up of large forest patches into smaller patches. Patch is a continuous area of a particular ecological community surrounded by distinctly different ecological communities [Baskent and Jordan, 1995], such as a forest continuous area surrounded by harvested lands or a clearcut surrounded by forestland. A clearcut is a patch where all trees were harvested. In a mature patch, the trees are older than a certain age. Within this age the trees are dominated by old growth conditions, such as large tree size, accumulation of large dead standing trees (snags), root and soil mounds, and nutrient cycling.

Example 1 Figure 1.1 provides an example of a forest of five mature stands. Supposing that stand B is harvested and two stands are adjacent if they share at least one single point, $\{B\}$ is a clearcut, and $\{A\}$, $\{C, D\}$ and $\{E\}$ are mature patches.

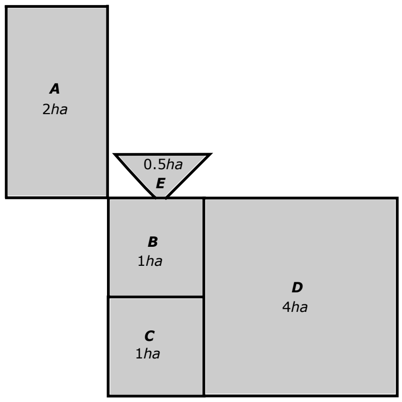

Figure 1.1: Five-stand mature forest.

■

From now on, forests referred to in this thesis are in old-growth or mature. These forests approximate the structure, composition, and functions of native forests and are related to the supply of wildlife habitat. They vary by forest type, but generally include more large trees, canopy layers, standing snags, native species, and dead organic matter than do young or intensively managed forests.

A mature patch can be characterized by spatial attributes like total area, interior or core area and proximity [Baskent and Jordan, 1995]. *Core area* is defined as the interior area of the patch where ecological functioning is free of the effect of surroundings conditions, the so-called *edge effect* [Franklin and Forman, 1987]. The edge effect corresponds to a buffer area (edge), where the environment differs significantly from the interior of the patch. *Proximity* of a patch concerns its spatial context in relation to the neighbors of the same type.

A *habitat* of an organism, population, or community is the geographically defined area where environmental conditions (e.g., climate, topography, etc.) meet the life needs (e.g., food, shelter, etc.). The habitat area in the forest is important, but also the representativeness of different habitat types, not only at the core area level but also at both core area and edge levels.

Core area provides habitat for wildlife to settle, reproduce, bite and sup, and it is preferred by specialist species [Hunter et al., 1990]. For example, most bird species are more dependent on core area than on the total area [Baskent and Jordan, 1995]. Some species depend on the more stable climatic environment of the forested interior, while others require the snags and decaying woody debris often found there. The importance of the size of these interior areas varies depending on the species. Some of the animals require large unbroken tracts of such habitat to hunt and breed. Terrestrial amphibian species may find adequate interior habitat conditions in considerably smaller forest patches.

Edge effects are both positive and negative. Edge effects may cause changes in the microclimatic conditions due to increased exposure to sunlight and wind. Some plants and animals benefit from the microclimatic edge effects, such as deer, moose, and elk [Bannerman, 1998]. Nevertheless, these edge effects may not favour many other species. For example, the physical damage caused by wind has been shown to reduce lichen abundance [Timonen, 2011]. The environmental conditions produced along the edges may modify habitat values that are important to interior forest dwellers. Furthermore, interior species may also be harmed through the ecological processes of predation, competition, and parasitism. When certain species migrate or disperse from edge to the interior, competition may increase and existing predator-prey and parasitic relationships may be altered [Bannerman, 1998].

Forest fragmentation diminishes both the quantity and quality of the core area at the expenses of the edge effects. The amount of core area is reduced since mature patches become smaller and the edge effects more pronounced. The quality is reduced as the smaller patches are affected by microclimatic and biotic edge effects [Harris, 1984, Bannerman, 1998].

The connectivity between habitats, related with the proximity of the habitats, is considered a key issue for the conservation of biodiversity and maintenance of natural ecosystem stability and integrity [Taylor et al., 1993]. Wildlife need to move between habitats. They need to access resources, ensure gene flow, shift their ranges, and establish new territories, among other things [Ament et al., 2014]. If the distances between the habitats are greater than particular species could travel, it is unlikely that the species will be able to persist.

Habitat connections depend not only on the travel distances that species need to cover to populate the habitats but also on the existence of units that shorten these distances, the intermediate stepping-stones (smaller mature patches) or wildlife corridors (vegetated linear strips, which differ from the surrounding land, connecting habitat patches). *Inter-habitat connectivity* can be set as the degree to which landscape promotes or prevents species movements among resource patches.

Another consequence of forest fragmentation is the isolation of some remaining interior spaces (increase of the distances between mature patches beyond the distances that species can travel), which can restrict the exchange of genetic material.

Many species have difficulty of survival in these modified environments of reduced size, new ecological edges and increased isolation.

All case study countries have developed policies or regulations on protected areas, *i.e.*, areas where harvesting, development and use are restricted by legal or other means for the conservation of nature. Protected areas may provide a fulfill range of functions, from habitats and inter-habitat connectivity to places of recreation that are important to many users.

A number of case study countries have developed initiatives to improve the representativeness of different habitat types. For example, in the state of Tasmania, in Australia, policies to maintain habitat diversity toward to the retention of wildlife habitats strips and patches containing trees with nesting hollows and other old growth structure elements in areas to harvest. In the province of New Brunswick, in Canada, six old-habitats were defined, based on the requirements of the vertebrate species assigned to them. Minimum requirements on habitats sizes were defined in terms of these requirements, ranging from 10 ha to 375 ha. In the United States of America, the 1973 US Endangered Species Act provides a program for protecting endangered species and their critical habitats. In Russian Federation, the new Forest Code of Russian Federation contains provisions for the protection of habitats of rare and endangered wildlife species, and the On Wildlife Act includes habitat protection and requires that any economic activity that impacts wildlife habitat must include mitigating measures. A key habitat is a usually concept in Sweden and Finland. It is a small habitat that is supposed to be particularly valuable for maintaining landscape-level biodiversity [Timonen, 2011, Simonsson, 2016]. They have an average size of 3.4 ha on private land and 8.0 ha on public land in Sweden, and 0.7 ha in Finland [Simonsson, 2016]. Natura 2000 [NATURA, 2017] is a network of nature protection areas in the territory of the European Union (EU). It stretches across all EU countries, including Finland, Sweden and Portugal. This network incorporates both Special Protection Areas (SPAs) under the EU Birds Directive (designated for the conservation of bird species including their habitat of European interest) and Special Areas for Conservation (SAC) under the EU Habitat Directive (designated for the conservation of habitats, and non-bird fauna and flora species of European interest). In Portugal, legislation has also been developed for the conservation of cork and holm trees.

Initiatives to maintain or increase the connectivity between protected areas are also considered by some countries studied. For example, in Canada, Quebec's new strategic guidelines in 2011 sets the importance of consolidating its network of protected areas by maintaining or improving connectivity between the different protected areas. The 2012 Planning Rule in the United States of America includes requirements for managing for ecological connectivity on national forest lands.

The main goal of this thesis is to develop mathematical models and methods in integer programming for solving harvest scheduling problems with environmental concerns. The research was structured in a collection of three papers, each one describing the study of a different forest management problem with respect to the environmental restrictions. In the last decades, several works have been done in this area, but as far as I know, none of them addresses all environmental restrictions considered in each problem.

All problems have non-spatial and spatial constraints. The non-spatial constraints are on the volume of timber harvested and average ending age of the forest. The volume constraints impose lower and upper bounds on the volume of timber harvested in each period. The average ending age constraint dictates a minimum average age for the forest at the end of the planning horizon. The spatial constraints are addressed to confront the environmental concerns. These concerns are on clearcut size, total habitat area, total core area and habitat connectivity, not all at the same time.

The problems of Papers 1 and 2 include a single objective, the maximization of the net present value, and the environmental concerns were all modeled by constraints. Nevertheless, specifying preferences for the requirement limits of some constraints before the solution process might be difficult. This aspect and the possibility of analyzing trade-offs between the net present value and other environmental objectives led to consider the third problem as a bi-objective harvest scheduling problem. In this problem, the two objectives are maximization of the net present value and the maximization of the inter-habitat connectivity.

Core area confounds multiple effects, and, as would be expected, there is no single measure that summarizes all these effects except core area itself. The core area of a patch is a function of patch size, shape and immediate surrounding conditions [Franklin and Forman, 1987, Baskent and Jordan, 1995].

Small mature patches have proportionally less amount of core area. As illustrated in figure 1.2, if a mature patch must be at least 100 metres from the boundary before it can be considered a core area, a four hectares square mature patch or a three hectares round mature patch contain no interior habitat (100% edge).

A circular mature patch would have to be almost eight hectares in size to contain just one hectare of interior habitat (87.5% edge approximately). Compact patch shapes should protect interior habitat against detrimental edge effects because these shapes have low edge-to-interior ratios. Conversely, convoluted and elongated patches have high edge-to-interior ratios. Thus, patches with complex shapes have proportionally more edge habitat than those of similar area but with compact shapes [Franklin and Forman, 1987].

Figure 1.3 shows that large mature patches do not necessarily provide more core area if their shapes are too elongated or complex, due to the edge effect.

With respect to immediate surrounding conditions, the amount of edge of a mature patch increases, and thus its core area decreases, with the contrast between the patch and these conditions.

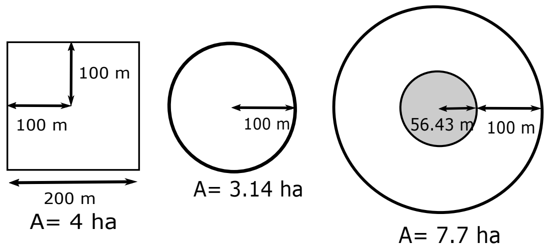

Figure 1.2: Core area (shaded area) and edge (white area) for three different patch sizes.

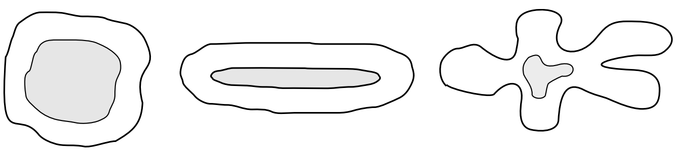

Figure 1.3: Core area (shaded area) and edge (white area) for three different patch shapes.

The three management problems are defined as follows:

Problem of Paper 1 - harvest scheduling problem with a single objective, the maximization of the net present value, non-spatial constraints and constraints on clearcut size, total habitat area and inter-habitat connectivity. For the sake of simplicity, it is considered that a habitat is a mature patch meeting a minimum target area. Core area is not addressed. As the inter-habitat connectivity is either cumbersome or difficult to measure and a connectivity index can capture the essential spatial changes within the forest with respect to this attribute, the connectivity was modeled through the index called *probability of connectivity* ([Saura and Hortal, 2007]).

Problem of Paper 2 - harvest scheduling problem with a single objective, the maximization of the net present value, non-spatial constraints and constraints on clearcut size, total habitat area and total core area. It is considered that a habitat is a mature patch with a minimum core area requirement. Inter-habitat connectivity is thus not addressed. The approach strategy for modeling core area is not the same as that used for the inter-habitat connectivity. Core area indices generally take into account the geometric shape of mature patches. Since there is no single indicator that summarizes all characteristics of shape and core area confounds effects other than shape, core area was directly measured using the concept of subregions [Wei and Hoganson, 2007, Zhang et al., 2011].

Problem of Paper 3 - harvest scheduling problem with two objectives, the maximization of the net present value and the maximization of the inter-habitat connectivity. Non-spatial constraints and constraints on clearcut size, total habitat area and total core area are addressed. It is considered that a habitat is a mature patch with a minimum core area requirement. Core area and inter-habitat connectivity are thus both addressed. The connectivity was modeled through the same index as in Paper 1, and core area was measured as in Paper 2.

The tree search methods branch-and-bound and Monte Carlo tree search were designed specifically to solve the problems, branch-and-bound for the problems of Papers 1 and 2, and Monte Carlo tree search for the problem of Paper 3. Both use a binary search tree of sequential decisions, where each decision corresponds to harvesting or not a stand in a given period. They could be used as heuristics, as a time limit of 2 hours was imposed.

All harvest scheduling problems were modeled in integer programming. The formulations are based on the so-called cluster formulation, one of the three basic formulations described in the literature for the harvest scheduling problems with maximum clearcut size constraints (*e.g.* Martins et al. [1999]). The reason for this choice is that the cluster formulation yields better linear programming bounds for these problems than the other two formulations [Goycoolea et al., 2009, Martins et al., 2012], the so-called path and bucket formulations, aspect that is crucial for the effectiveness of branch-and-bound.

Relatively to the problem of Paper 1, connectivity constraints impose a minimum for the connectivity index in each period. The index selected satisfies a set of properties that any ideal connectivity index should fulfill. In addition, its value in the branch-and bound tree does not increase from a node to its children nodes (which will be proved in Appendix C), an important property for the implemented branch-and-bound. The index, used also in Paper 3, is based on the habitat availability concept and dispersal inter-patch distances, and uses graph structures.

A graph was used to represent the forest region, were each vertex is associated to a stand and an edge between two vertices reflects the adjacency between the corresponding stands. Clearcuts and mature patches are represented by connected components in the graph whose vertices correspond to the harvested stands and mature stands, respectively. In the context of the connectivity index, a graph simplifies a forest down to a network of vertices that represent the mature patches.

Branch-and-bound for the problems of Papers 1 and 2 was proposed for the following reasons. This tree search method is by far the most widely used tool for solving exactly models in linear integer programming. As an exact method, has the potential of finding optimal solutions, if enough time is available. If it is stopped after some prescribed time or when other criteria are satisfied, it provides a bound on the value of the optimal solution, thus measuring the quality of the solution obtained.

However, the formulation proposed in Paper 1 is in non-linear integer programming, as the connectivity constraints are non-linear, which precludes the direct use of general purpose solvers for linear integer programs. Even if the connectivity constraints were linear, the number of variables was extremely large for large instances, which would also preclude the direct use of such solvers. For these reasons, a branch-and-bound procedure, specific for the problem, was implemented.

The formulation proposed in Paper 2 is in integer linear programming, but the number of variables is also extremely large for large instances. As for the first problem and with the instances used in this thesis, the implemented branch-and-bound was able to find very good solutions (within 1% of the optimum) or the optimum, adopting this framework to the new problem seemed to be a good strategy.

In Paper 3, a multi-objective Monte Carlo tree search was developed to find a subset of efficient solutions for the bi-objective harvest scheduling problem. This method was used as an alternative to standard binary tree search, once the construction and storage of the tree is computationally expansive, primarily for medium and large instances. Monte Carlo tree search builds a tree in an incremental and asymmetric manner that adapts to the topology of the search space. It visits more interesting nodes more often, and focuses its search time in more relevant parts of the tree.

The performances of the branch-and-bound and Monte Carlo tree search methods were tested with forest instances available in website http://www.unbf.ca/fmos/ (Integrated Forest Management Lab 2006). There were tested sixteen different instances, ranging in size, from small to large, inducing from relatively easy to hard models to solve. The instances include *El Dorado*, a National forest in northern California (referred to in Goycoolea et al. [2005, 2009], Könnyű and Tóth [2013]), *Stafford*, a forest in British Columbia [Crowe et al., 2003], *Kittaning*4, *Bear Town*, *PhyllisLeeper*, and *FivePoints*, forests in Pennsylvania, USA [Könnyű and Tóth, 2013], *WLC* [Bettinger et al., 2002], and the computer generated instances (through the Forest Landscape Generator) *FLG*9 and *FLG*10 [Paradis and Richards, 2001].

The number of stands ranges from 32 (Kittaning4) to 1363 (El Dorado). All stands are characterized by the area and age, together with timber attributes, such as the volume of timber produced and the net present value generated by the harvest of this volume. Some instances were tested with a different number of periods, from three to nine periods in the problem of Paper 1 and from three to eight in the other problems. For the sake of simplicity, it is assumed that the planning horizon is such that a stand can be harvested at the most once, *i.e.*, the minimum rotation in the stand is longer than the planning horizon.

The remaining of this thesis is organized as follows. Chapter 2 reviews the literature in the context of operations research in forest management with environmental restrictions.
Chapter 3 , chapter 4 and chapter 5 set the three papers in order of publication.
Chapter 6 summarizes the research conclusions and proposals for future investigations.
Appendix A describes in more detail the forest policies and laws that have emerged to address environmental concerns in the seven countries selected.
Appendix B comprises the methods used throughout the thesis.
Appendix C highlights some properties of the connectivity index.
Appendix D presents the procedures and algorithms used by branch-and-bound and Monte Carlo tree search.

Chapter 2

Literature review

This section aims at providing a literature review in the context of operations research in forest harvest scheduling problems with environmental concerns.

Forest harvest scheduling problems have been developed to deal with management policies and laws. With the intention of reducing the environmental impacts of harvest schedules, maximum clearcut size constraints have been incorporated into the planning process. Murray [1999] identified two approaches for modeling forest harvest scheduling problems with this type of constraints. The first is the classic approach, in which stands are aggregated and redefined so that any two adjacent new stands exceed the maximum clearcut area. Murray referred to this approach as the *Unit Restriction Model* (URM). The key to the URM being applicable is that stand sizes are defined appropriately (*e.g.*, stands are 25-49 ha in size if the maximum clearcut area is 49 ha). The second approach, the Area Restriction Model (ARM), assumes that the stands are substantially smaller than the maximum clearcut area, so harvesting simultaneously two adjacent stands in the same period does not necessarily represent a violation (*e.g.*, if stands range between 10 and 25 ha in size and the maximum clearcut area is 49 ha, there could potentially be up to four contiguous stands simultaneously harvested). Murray and Weintraub [2002] showed that the ARM significantly improves the quality of the solutions, in view of its greater flexibility. Nevertheless, it is substantially more difficult to solve, as it is more combinatorial.

Three main basic integer programming models have been proposed in the literature for the ARM: the *path* formulation [Martins et al., 1999, McDill et al., 2002, Murray and Weintraub, 2002, Crowe et al., 2003, Könnyű and Tóth, 2013] defines a binary variable for each stand and period, that takes the value 1 if the stand is harvested in the period and 0 otherwise, and constraints that prohibit that the area of each clearcut does not exceed the threshold value (feasible clearcut); the *cluster* formulation [Martins et al., 1999, McDill et al., 2002, Martins et al., 2005, Goycoolea et al., 2005, Vielma et al., 2007, Martins et al., 2012] defines a binary variable for each feasible clearcut and period, that takes the value 1 if the clearcut is selected in the period and 0 otherwise, and constraints that prevent any selected pair of clearcuts from being overlapping or adjacent; the *bucket* formulation [Constantino et al., 2008, Goycoolea et al., 2009, Martins et al., 2012] defines a binary variable for each stand

and period, that takes the value 1 if the stand is assigned to a bucket, empty a priori, in the period, and 0 otherwise, and constraints that prohibit a stand from being assigned to more than one bucket and non-empty buckets from being adjacent. The main drawbacks of the first two models are, respectively, the large number of constraints and the large number of variables. These numbers may grow exponentially with the number of stands. The third model has a polynomial number of variables and constraints.

Goycoolea et al. [2009] have compared the three formulations from a computational standpoint and they have proven that the cluster formulation yields better linear programming bounds than the path formulation and they have shown that there is no dominance relationship between the bounds of the bucket and path formulations. Martins et al. [2012] have proven that the cluster formulation dominates the bucket formulation. The properties of these different formulations can significantly affect the performance of the solution methods, in particular the branch-and-bound.

Most of the large linear integer programming problems are notoriously difficult to solve using exact algorithms, and only small to medium-sized problems have thus far been solved to optimality with the direct use of general purpose solvers for this type of problems. The limitations of integer programming to incorporate spatial constraints for large forests and over long time horizons led to other direction of research, the use of heuristics. A heuristic is a search algorithm that does not necessarily find the optimal solution but it can produce relatively good solutions within reasonable time frames.

Different heuristics have been reported for both the URM and ARM approaches, as tabu search (Richards and Gunn [2000], Boston and Bettinger [2002], Caro et al. [2003], Richards and Gunn [2003] for the ARM), simulated annealing (Nelson and Brodie [1990] for the URM; Lockwood and Moore [1993], Falcão and Borges [2002] for the ARM), hybrid heuristics (Falcão and Borges [2002] for the ARM), genetic algorithms (Boston and Bettinger [2002] for the ARM), and column generation (Weintraub et al. [1994] for the URM; Martins et al. [2005] for the ARM).

Forcing harvests into small sizes is a strategy that will likely fragment the forest. Often mature patches have become too small to be suitable for many species and the distances between them may be greater than the species could travel. The issue of fragmentation has been addressed into forest harvest scheduling problems adding substantial complexity to the models and solution techniques.

One way to mitigate the fragmentation effect is to consider mature patch area constraints, *i.e.*, constraints that require a minimum area for each mature patch [Öhman and Lämås, 2005, Öhman and Wikström, 2008]. These constraints require the merging of mature patches. [Hof et al., 1994] modeled indirectly habitat fragmentation using mixed integer linear programming formulations that focus on wildlife growth and dispersion as a dynamic and a probabilistic processes. Some studies included both constraints on clearcut and mature patch sizes, using integer programming [Martins et al., 1999, Rebain and McDill, 2003a,b, Martins et al., 2005] or heuristics (simulated annealing and hybrid heuristics by Falcão and Borges [2002], tabu search by Caro et al. [2003]). All these studies modeled clearcut size constraints in the context of the ARM.

While mature patch area constraints can create large mature patches, these patches may have a small amount of core area if they are elongated or irregularly shaped. This underscores the importance of also integrating the core area issue in forest harvest scheduling problems with fragmentation concerns.

Core area can be modeled either directly or indirectly. Approaches that include core area have been developed in some studies using integer programming (Wei and Hoganson [2007] and Zhang et al. [2011], where core area was modeled directly using the concept of subregions) and heuristics (simulated annealing by Öhman and Eriksson [1998], Öhman [2000], Öhman et al. [2002] and dynamic programming-based heuristics by Hoganson et al. [2005], Wei and Hoganson [2008]). None of these studies considered constraints on clearcut area.

Indirect approaches used patch area and shape that together with the immediate surrounding conditions, determine the amount of core area, and can be used to approximate core area. Öhman and Lämås [2005] used a bi-objective problem where the minimization of the shape index [McGarigal et al., 2002] was regarded as an additional objective besides the maximization of the net present value. Clearcut size constraints are also not considered in this work.

There are many other approaches that tried to capture the essential characteristics of shapes [Davis, 1977, Moellering and Rayner, 1981, Xia, 1996, Wentz, 2000]. Measures of perimeter [Öhman and Wikström, 2008, Tóth and McDill, 2008] and area [Martins et al., 1999, Falcão and Borges, 2002, Caro et al., 2003, Rebain and McDill, 2003a,b, Martins et al., 2005, Tóth et al., 2006] could also be considered indirect approaches to model core area. Öhman and Wikström [2008] considered a bi-objective problem where the minimization of the total perimeter of the mature forest and the maximization of the net present value are the objectives, and clearcut size constraints are not addressed. Tóth and McDill [2008] minimized the total perimeter of the mature forest with constraints on clearcut size, where these constraints are modeled as ARM.

The loss of habitat connectivity resulting from fragmentation is a major threat for wildlife dispersal and survival, and for the conservation of biodiversity in general. A frequently cited recommendation for protecting biodiversity is the improvement of habitat connectivity to ensure that species can move and adapt in response to climate induced changes [Ament et al., 2014].

Different approaches have been suggested for landscape conservation planning with the main objective of improving the connectivity. Some approaches used wildlife corridors, which are important to link areas of habitat and facilitate movement. For example, Williams [1998] incorporated spatial connectivity into a bi-objective binary programming model by specifying sets of stands that were not going to be harvested, which would then be corridors between areas that had been assigned to wildlife. The objectives are the minimization of corridor land costs and the minimization of the amount of unsuitable land within the corridor system.

Hof et al. [1994] modeled connectivity indirectly proposing formulations that take into account the spatial and temporal developments of wildlife populations. Other approaches used indices that try to capture the impact of landscape changes with an emphasis on the underlying processes.

Different indices have been suggested for measure the connectivity. Shumaker [1996] proposes an index based on the perimeter and area of each mature patch. The majority of the indices are based on the pairwise connections between the mature patches, which can be measured in different ways: by the distance between the patches [McGarigal et al., 2002], being one when the distance between two patches is not greater than the maximum dispersal distance or null otherwise [Keitt et al., 1997, Hortal and Saura, 2006]; by some function that reflects the possibility of dispersal at a given distance [Bunn et al., 2000, Saura and Hortal, 2007, Neto et al., 2013].

Single-objective models have often been applied to solve forest management problems expressing the environmental concerns by constraints. These constraints impose limits on, for example, habitat area, core area and total perimeter of mature forest. Before the solution process, it might be difficult to identify the appropriate threshold requirement limits that adequately meet the environmental concerns. Beyond that, sometimes timber and environmental objectives are in conflict with each other, and a multi-objective approach is more appropriated.

Snyder and ReVelle [1997] used a bi-objective integer programming model that addresses the management of two conflicting objectives, which are related respectively with the wildlife habitat and timber harvesting activities. Timber harvest constraints were incorporated into the model to prevent harvests from occurring next to one another, as well as next to habitat areas.

Multi-objective models have been applied to forest management problems, in general, by transforming the objectives into a weighted-sum single-objective function. These models were solved by exact methods [Snyder and ReVelle, 1997, Williams, 1998] or heuristics [Öhman and Eriksson, 1998, Williams, 1998, Öhman, 2000, Öhman et al., 2002, Öhman and Lämås, 2005, Öhman and Wikström, 2008].

Tóth et al. [2006] evaluate the performance of five traditional methods including a new proposed method, the so-called *alpha-delta*, to generate the efficient frontier for a bi-objective harvest scheduling problem, where the objectives are the maximization of the net present value and the maximization of the total area of forest patches with a minimum area requirement. In this work, clearcut size constraints were addressed in the context of the ARM. Tóth and McDill [2008] compared a single objective model that minimizes the total perimeter of the mature forest with the previous bi-objective model. In this work, the bi-objective model was solved by the alpha-delta method.

Several of the above mentioned problems were formulated in integer programming and solved exactly using the branch-and-bound method. Integer programming has been widely and successfully used in combinatorial optimization. The branch-and-bound method was first proposed by Land and Doig [1960] and it is the most widely used method for solving large scale $\mathcal{NP}$-hard combinatorial optimization problems. For example, Clausen [1997] gives an overview of the application of the method for the Traveling Salesman, Graph

Partitioning and Quadratic Assignment problems. An advantage of branch-and-bound is that it can be used as an exact method (the size of the problems that can be solved exactly is generally smaller) or, if it is interrupted, as a heuristic [Pedroso and Rei, 2015]. In the latter case, the procedure is stopped when the solution is considered satisfactory, or a time limit is reached.

Monte Carlo methods have applications in numerous fields. Monte Carlo integer programming is a heuristic commonly used in forest management problems, particularly for URM problems. This method examines randomly generated harvest schedules, preserving good feasible schedules. It was applied, for example, in Nelson and Brodie [1990], to solve a combined harvest scheduling and transportation planning problem with constraints on clearcut area, Clements and Dallain [1990], to integrate a randomly generated harvesting sequence with harvest-flow and constraints on adjacent stands and O'hara et al. [1989], to solve a spatial harvest scheduling problem. The three studies included constraints on clearcut area modeled as URM.

Monte Carlo tree search (MCTS) was proposed by Coulom [2006] and applied with considerable success to the game of Go (9×9 board). Game-playing is still the area where the method and its many variants are most commonly used. In recent years, MCTS has been growing in popularity and has also been applied ín other domains. There are, however, very few publications in combinatorial optimization. Sabharwal et al. [2012] and Pedroso and Rei [2015] used MCTS to solve specific optimization problems. At far as I known, there are no applications of MCTS to forest management problems and the only application of MCTS to the multi-objective optimization has been performed by Wang and Sebag [2012].

Chapter 3

A branch-and-bound procedure for forest harvest scheduling problems addressing aspects of habitat availability

This chapter is composed of the first of three papers co-authored by the PHD candidate. The paper was published in the International Transactions in Operational Research [Neto et al., 2013].

A branch-and-bound procedure for forest harvest scheduling problems addressing aspects of habitat availability

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Abstract

In the literature, the most widely referred to approaches regarding forest harvesting scheduling problems involving environmental concerns have typically addressed constraints on the maximum clearcut area. Nevertheless, the solutions arising from those approaches in general display a loss of habitat availability. That loss endangers the survival of many wild species. This study presents a branch-and-bound procedure designed to find good feasible solutions, in a reasonable time, to forest harvest scheduling problems with constraints on the clearcut area and habitat availability. Two measures are applied for the habitat availability constraints: the area of all habitats and the connectivity between them. In each branch of the branch-and-bound tree, a partial solution leads to two children nodes, corresponding to the cases of harvesting or not harvesting a given stand in a given period. Pruning is based on constraint violations or unreachable objective values. Computational results are reported.

1 Introduction

Forest management models for timber production have been addressing concerns related to resources other than timber, such as wildlife, soil, water and aesthetics values. Modeling approaches to confront these concerns have mainly involved the use of restrictions that dictate a maximum in the area of each clearcut (continuous harvested region). However, the solutions arising from these approaches typically display a dispersion of smaller clearcuts across the forest and thus, a fragmented forest. It is well known that fragmentation of a mature forest may have significant negative impacts on some wildlife species. It leads to a reduction in habitat availability, that is, the habitat area shrinks and the inter-habitat connections weaken (Harris, 1984; Kurtilla et al., 2002).

Mature patches are related with the supply of wildlife habitat (Franklin and Forman, 1987). A patch is a group of contiguous stands distincts from its surroundings. A stand is an ecologically homogeneous unit resulting from the classification of the landscape for forest management purposes. A mature patch is a forest patch that is older than a certain age. Some animal species are more dependent on the interior or core area of a mature patch, or on the contrary on its edge, rather than on its total area (Baskent and Jordan, 1995). For the sake of simplicity, in this study we consider a habitat (for certain wildlife species living in a mature forest) as a mature patch meeting a minimum target area.

Connectivity has been described as the degree to which landscape promotes or prevents species movements among resource patches. It is considered a key issue for conservation of biodiversity and the maintenance of natural ecosystem stability and integrity (Taylor et al., 1993). It depends on the travel distances that species need to cover to populate the habitats and on the existence of intermediate steps (*e.g.*mature non-habitats) or corridors (*e.g.*strips of mature forest) that shorten these distances.

One way to handle habitat availability in forestry planning is to include mature patch area constraints (Öhman and Lämås, 2005). A mature patch area constraint requires that a certain amount of the forest encompasses continuous blocks of mature forest. Forest harvest scheduling problems addressing this issue have been used in several studies, using exact integer programming approaches (Hof et al., 1994; Martins et al., 1999; Rebain and McDill, 2003a and 2003b; Tóth et al., 2006; Wei and Hoganson, 2007; Öhman and Wikström, 2008) or heuristics (Öhman and Eriksson, 1998; Öhman, 2000; Falcão and Borges, 2002; Öhman, 2002; Caro et al., 2003; Hoganson et al., 2004; Martins et al., 2005; Öhman and Lämås, 2005; Mathey et al., 2005; Wei and Hoganson, 2008). However, these approaches can create large habitat patches, but do not necessarily create the conditions for individuals to move among them. The objective of including connectivity in this study is to reduce the hindrance of wildlife movement among habitats, or in other words, to increase the inter-habitat connectivity. We use an index for this purpose. Several connectivity indices have been proposed for landscape conservation planning, see *e.g.*Fragstats (1995), Shumaker (1996), Keitt et al. (1997), Bunn et al. (2000), Hortal and Saura, (2006), Saura and Pascual-Hortal (2007). We use the index proposed in this last study, the so-called *probability of connectivity*. To date, as far as we know, no method for harvest scheduling problems that explicitly address the inter-habitat connectivity issue has been reported.

Exact integer programming approaches have an advantage over heuristics when full search is possible in reasonable time, as they determine proved optimal solutions. When the problems are too large to be solved exactly, exact methods may be interrupted in the middle of the search. Methods based on linear programming (*e.g.*branch-and-bound) provide a measure of the quality of the best known solution in each step, as well as a bound to its value. In this way, they can be stopped if this solution is satisfactory. A branch-and-bound procedure can be used as an exact method, especially when it comes to solving academic problems (Sbihi, 2007; Artigues et al., 2009) or, if it is interrupted, as a heuristic (Pedroso and Kubo, 2010). In the latter case, the procedure is stopped when the solution is considered satisfactory, or a time limit is reached.

In this work, we present a branch-and-bound approach to find good feasible solutions, in a reasonable time, to forest harvest scheduling problems with constraints on clearcut area and on both total habitat area and inter-habitat connectivity. In sections 2 and 3, we describe and formulate the forest harvest scheduling problem studied. In section 4, we present the branch-and-bound procedure. In section 5, we report on computational experience. The tests were performed with forests ranging from 32 to 1363 stands. In the last section, we present some conclusions.

2 Problem definition

Forest harvest scheduling problems typically deal with determining which stands should be harvested in each period during a planning horizon in order to maximize the net present value of the timber harvested. The optimization model proposed here considers two main types of constraints, namely spatial constraints (affecting the relative arrangement of stands and the interconnections among them) and non-spatial constraints. In this study, as non-spatial constraints, we consider lower and upper bounds on the volume of timber harvested and a minimum average age for the forest at the end of the planning horizon. Volume constraints ensure a regular production of timber, mainly to guarantee that the industry is able to continue operating with similar levels of machine and labor utilization. The ending-age constraint prevents the model from over-harvesting the forest (Tóth et al., 2006). As spatial requirements, we consider constraints on the clearcut area and on habitat availability. Constraints on clearcut area impose a maximum in the area of each clearcut and a minimum number of periods in which stands adjacent to a clearcut cannot be harvested, the so-called *greenup* restrictions. Constraints on habitat availability impose, in each period, a minimum in the total area of habitats and a minimum value for the connectivity index, described in more detail below. We shall refer to this problem as P_0. It is assumed the horizon is such that each stand may be harvested at most once. Minimum harvest ages are considered. It is also assumed that a harvested stand may become mature within the time horizon.

We use a connectivity index to quantify the inter-habitat connectivity. The authors in Saura and Pascual-Hortal (2007) encourage the use of the *probability of connectivity* index as a sound basis on which to plan decision-making. They use an indicator p_{lm} of the possibility of a direct movement occurrence of an animal (without considering passing through any intermediate mature patch) between mature patches l and m. These mature patches also meet a minimum required area which can be less than the minimum habitat area if they are intermediate steps or corridors. Indicator p_{lm} is obtained by the negative exponential function

$$p_{lm} = e^{-Cd_{lm}}, \tag{1}$$

where C is a constant greater than zero called the coefficient of dispersion (species' dependent), and d_{lm} is the edge-to-edge distance between l and m. For a particular wildlife species, constant C is computed by solving equation (1) in order to C where d_{lm} and p_{lm} are replaced, respectively, by a specific distance between l and m and the expected indicator value for this distance. The closer the indicator is to 1, the smaller the inter-mature patch distance, and the more favorable the occurrence of a direct movement.

A path between two habitats h and r is made up of a sequence of direct movements from h to r in which no intermediate mature patch is visited more than once. The *connectivity of a path* is given by the product of the indicators of direct movements that form the path. The largest connectivity among all paths between h to r is denoted by g_{hr}, and indicates the path with the greatest chance of dispersion. Observe that the path with largest connectivity corresponds precisely to the shortest distance path.

Let $\mathcal{H}_t$ be the set of all habitats in period t, s_h be the area of habitat h for all $h \in \mathcal{H}_t$ and F be the total area of the forest. The connectivity index for period t is given by

$$I_t = \frac{\sum_{h\in\mathcal{H}_t} \sum_{r\in\mathcal{H}_t} \mathsf{s}_h \mathsf{s}_r \mathsf{g}_{hr}}{\mathsf{F}^2}. \tag{2}$$

I_t expresses the possibility of two animals randomly placed in the forest falling into interconnected habitats. I_t ranges from 0 to 1, and increases with connectivity improvement. It is equal to 1 when all the forest is a single habitat, and is equal to zero when there are no habitats, or all habitats are completely isolated (by being too distant).

3 Model

Here we present a mathematical optimization model for the harvesting scheduling problem described in the previous section (problem P_0). The model is an extension of the integer programming model referred to in the literature as the cluster formulation. The introduction of constraints on the connectivity index makes the model non linear.

Three main basic integer programming models for the harvest scheduling problem with constraints on the maximum clearcut area have been described in the literature. The path formulation (Martins et al., 1999; McDill et al., 2002; Murray and Weintraub, 2002; Crowe et al., 2003) encompasses an exponential number of constraints. The cluster formulation (Martins et al., 1999; McDill et al., 2002; Martins et al., 2005; Goycoolea et al., 2005, Vielma et al., 2007) encompasses an exponential number of variables. The most recent bucket formulation (Constantino et al., 2008) has a polynomial number of variables and constraints. It has proven theoretically that the cluster formulation dominates both the path formulation (Goycoolea et al., 2009) and the bucket formulation (Martins et al., 2012). In other words, the LP bound of the cluster formulation is tighter than those of the other two formulations. As the branch-and-bound described in this work uses LP bounds of the harvest scheduling problem with constraints on the maximum clearcut area, we propose to model P_0 using the cluster formulation.

To identify potential clearcuts or mature patches one must define adjacency between stands. For clearcuts, we consider that two stands are adjacent if they share a boundary with positive length, *i.e.*, that is not a discrete set of points. For mature patches, we consider that it is sufficient to share at least a single point. In Goycoolea et al. (2005), the first and second definitions are referred to as *strong* and *weak* adjacency, respectively.

Let $G = (\mathcal{V}, \mathcal{E})$ be a graph, where each vertex in $\mathcal{V} = \{1, \ldots, n\}$ corresponds to a stand of the forest and the endpoints of each edge in $\mathcal{E}$ correspond to two adjacent stands according to the definition of strong adjacency. As a consequence, the graph is planar, *i.e.*it can be drawn in a plane surface without crossing edges. Let $\mathcal{T} = \{1, 2, \ldots, T\}$ represent the planning horizon. Let $\mathcal{K}$ be the set of all subsets of vertices that generate maximal cliques. A clique is a complete subgraph of the graph, *i.e.*it has an edge between each pair of vertices, and it is maximal if it is not contained in any other clique. Since the graph is planar there are no cliques with more than four vertices.

Before presenting the mathematical formulation, we define the following notation:

$\mathsf{A}^{\mathsf{max}}$ - maximum clearcut area;

$\mathsf{H}^{\mathsf{min}}$ - minimum habitat area;

$\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$ - minimum total habitat area in each period;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$ - minimum harvest age;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ - minimum mature age;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$ - minimum average age of the forest at the end of the planning horizon (in periods);

$\mathcal{C}_t$ - set of all potential clearcuts in period t, that is, all possible continuous regions such that the area is less than or equal to $A^{\max}$ and all stands in t are not less than $\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$ old;

$\mathcal{H}'_t$ - set of all potential habitats in period t, that is, all possible continuous regions such that the area is not less than $\mathsf{H}^{\mathsf{min}}$ and all stands in t are not less than $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ old;

s_c - area of region c;

npv_{ct} - net present value of timber provided by region $c \in \mathcal{C}_t$ if it is harvested in period t;

v_{ct} - volume of timber provided by region $c \in \mathcal{C}_t$ if it is harvested in period t;

v_0 - target volume of timber to be harvested in each period;

α - deviation allowed from target volume of timber to be harvested;

G - number of green-up periods;

I - minimum value for the connectivity index in each period.

The decision variables are the following:

$$z_{ct} = \begin{cases} 1 & \text{if region } c \in \mathcal{C}_t \text{ is harvested in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ht} = \begin{cases} 1 & \text{if region } h \in \mathcal{H}'_t \text{ is habitat in period } t \\ 0 & \text{otherwise.} \end{cases}$$

Since the connectivity index is non-linear, the formulation of P_0 is a non-linear integer program:

$$\max \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}_t} \mathsf{npv}_{ct} z_{ct} \tag{3}$$

subject to

$$\sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{C}_t:\, i\in c} z_{ct} \le 1,\ \forall i \in \mathcal{V} \tag{4}$$

$$\sum_{c\in\mathcal{C}_t} \mathsf{v}_{ct} z_{ct} \ge (1-\alpha)\mathsf{v}_0,\ \forall t \in \mathcal{T} \tag{5}$$

$$\sum_{c\in\mathcal{C}_t} \mathsf{v}_{ct} z_{ct} \le (1+\alpha)\mathsf{v}_0,\ \forall t \in \mathcal{T} \tag{6}$$

$$(T+1)\mathsf{F} - \sum_{t\in\mathcal{T}} t \sum_{c\in\mathcal{C}_t} \mathsf{s}_c z_{ct} \ge \mathsf{Age}_{\mathsf{end}}^{\mathsf{min}}\mathsf{F} \tag{7}$$

$$\sum_{u=t}^{t+\mathsf{G}-1} \sum_{c\in\mathcal{C}_t:\, c\cap q\neq\emptyset} z_{cu} \le 1,\ \forall q \in \mathcal{K},\ \ t = 1,\dots,T-(\mathsf{G}-1) \tag{8}$$

$$\sum_{u=max\{1,t-(\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}}-1)\}}^{t} \sum_{c\in\mathcal{C}_t:\, i\in c} z_{cu} + \sum_{h\in\mathcal{H}'_t:\, i\in h} y_{ht} \le 1, \forall i \in \mathcal{V},\ \forall t \in \mathcal{T} \tag{9}$$

$$\sum_{h\in\mathcal{H}'_t} \mathsf{s}_h y_{ht} \ge \mathsf{H}_{\mathsf{tot}}^{\mathsf{min}},\ \forall t \in \mathcal{T} \tag{10}$$

$$\frac{\sum_{h\in\mathcal{H}'_t} \sum_{r\in\mathcal{H}'_t} \mathsf{g}_{hr} \mathsf{s}_h y_{ht} \mathsf{s}_r y_{rt}}{\mathsf{F}^2} \ge \mathsf{I}, \forall t \in \mathcal{T} \tag{11}$$

$$z_{ct} \in \{0,1\},\ \forall c \in \mathcal{C}_t,\ \forall\, t \in \mathcal{T} \tag{12}$$

$$y_{ht} \in \{0,1\},\ \forall h \in \mathcal{H}'_t,\ \forall\, t \in \mathcal{T}. \tag{13}$$

Expression (3) maximizes the net present value of timber harvested. Constraints (4) ensure that each stand is harvested once at the most in the horizon. Constraints (5) and (6) require minimum and maximum volumes of timber harvested in each period, respectively. Constraint (7) requires the average age of the forest at the end of the planning horizon, in period $T+1$, to be at least $\mathsf{Age}_{\mathsf{end}}^{\mathsf{min}}$ periods. Constraints (8) guarantee that if a potential clearcut is harvested in a certain period then no adjacent potential clearcuts are harvested in that period and during the green-up time. Constraints (9) ensure that each stand belongs in the utmost to one region selected either to be harvested or to be a habitat. For each period, constraints (10) and (11) impose a minimum total habitat area and a minimum value for the connectivity index, respectively. Constraints (12) and (13) state the binary nature of the decision variables.

4 Branch-and bound

Here we develop a branch-and-bound procedure designed specifically for solving the model presented in the previous section. Two main reasons lead us to choose this approach. On the one hand, the model includes both non linear constraints and exponentially many variables, precluding the use of general purpose solvers. On the other hand, branch-and-bound, as an exact method, has the potential of finding optimal solutions, if enough time is available. It also can be used as a heuristic if it is stopped after some prescribed time or when other criteria are satisfied. A bound on the value of the optimal solution is also provided, giving some measure of quality of the solution obtained.

The branch-and-bound procedure consists of successive branching on partial solutions of P_0. These solutions have a correspondence to z_{ct} and y_{ht} of P_0, but their representation is different. More specifically, in each branch a partial solution can lead to two children solutions, corresponding to the decisions of harvesting or not a given stand in a given period. Node pruning is based on constraint violations or unreachable objective values.

At each node k, solution x^k is an integer vector with n components, where x_i^k is the period in which stand i is harvested. If the stand is not harvested across the planning horizon, $x_i = T + 1$. Let $\mathrm{fnpv}(x^k)$ be the net present value of x^k and ub^k an upper-bound on the optimal net present value of P_0 considering the decisions taken at k and its predecessor nodes.

Branch-and-bound makes use of a queue Q of nodes to explore. The first step is to initialize queue Q with the root node, defined by the following elements:

- set $\mathcal{S}_0$ of all pairs $(\text{stand } i, \text{period } t)$ such that i is available to be harvested in t, sorted by descending order of the net present value corresponding to i and t;
- solution x^0 where no decision is taken ($x_i^0 = T + 1$ for all stands i);
- upper bound ub^0, described further.

The maximum cardinality of $\mathcal{S}_0$ is $n \times T$, which occurs when all stands are old enough to be harvested in any period.

At each node k, the first element (i_k, t_k) of $\mathcal{S}_k$ is selected. The partial solution x^k leads to two new partial solutions, corresponding to the decision of harvesting or not stand i_k in period t_k (left and right branches, respectively):

- x^{k+1}, were we fix $x_{i_k}^{k+1} = t_k$ and $x_i^{k+1} = x_i^k$ for all $i \neq i_k$;
- x^{k+2}, with $x_i^{k+2} = x_i^k$ for all i.

The sets $\mathcal{S}_{k+1}$ and $\mathcal{S}_{k+2}$, corresponding to the two new branches, are initialized by removing (i_k, t_k) from $\mathcal{S}_k$. Set $\mathcal{S}_{k+1}$ is updated by removing the remaining pairs that contain i_k. It is also removed any pair (i, t) such that harvesting stand i in period t violates the following constraints: upper bound on the volume of timber harvested in each period; ending-age constraint; maximum clearcut area and greenup constraints if stands i and i_k are adjacent. This contributes to a significant reduction of the branch-and-bound tree, which is attractive especially because the cost of checking these constraints is low.

At any node k', the constraints of lower bound on the volume harvested in each period can only be fully checked if $\mathcal{S}_{k'}$ is empty (all the decisions have been taken). In this case, if the corresponding solution $x^{k'}$ does not satisfy that constraints, k' is infeasible, otherwise k' is feasible. However, when $\mathcal{S}_{k'}$ is not empty, we check for period $t_{k'}$, and in the green-up time in left branches, if harvesting all stands still available gives a volume of timber greater than or equal to the lower bound (infeasibility test). If not, node k' is infeasible, as it cannot lead to solutions meeting the lower bound of volume constraints. Otherwise, if constraints on total habitat area and inter-habitat connectivity are satisfied, no conclusion is drawn as to the infeasibility of k', since node k' can lead to solutions satisfying all constraints.

We check nodes $k+1$ and $k+2$ using the infeasibility test. If we do not conclude that node $k+1$ is infeasible, more updates are made: $\mathrm{fnpv}(x^{k+1})$ is equal to $\mathrm{fnpv}(x^k)$ plus the net present value of stand i_k in period t_k, and upper-bound ub^{k+1} is calculated (see Section 4.1). If no conclusion is drawn about the infeasibility of $k+2$, $\mathrm{fnpv}(x^{k+2}) = \mathrm{fnpv}(x^k)$ and upper-bound ub^{k+2} is calculated. Whenever a solution is feasible, its value is compared with the best solution value obtained so far (the *incumbent*). If a better value is obtained the incumbent is updated.

Any node k' can be pruned for one of the following three reasons:

- $x^{k'}$ is feasible and $\mathcal{S}_{k'}$ is empty; in this case the incumbent is updated if $\mathrm{fnpv}(x^{k'})$ is larger;
- k' is infeasible (either $\mathcal{S}_{k'}$ is empty or not);
- upper-bound $ub^{k'}$ is not greater than the best net present value found so far.

The new (non-pruned) nodes are inserted into queue Q and the process continues from these elements. Branch-and-bound ends when Q is empty (no nodes to explore), or a certain CPU time limit is reached.

The method can be represented by a tree, as shown in Figure 1. The tree has a maximum height of $n \times T + 1$ and a maximum number of nodes of $2^{n \times T+1} - 1$.

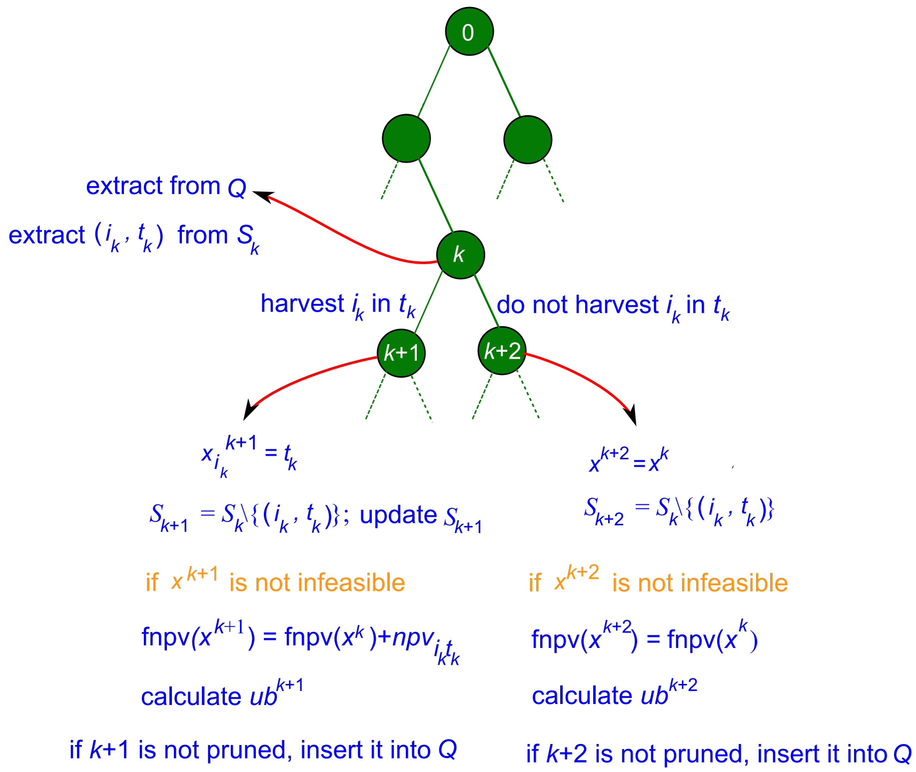

Figure 1: Branch-and-bound.

4.1 Upper-bounds

Upper-bounds on the optimal value are used to prune nodes of the branch and bound tree. Indeed, when the upper bound associated to a node is less than or equal to the incumbent, no improvement on the solution value is possible in that node.

We tested five possibilities for computing upper-bounds to the optimal net present value of problem P_0. Let ub_j with $j = 0, \cdots, 4$ denote these upper-bounds. Only one upper-bound type is used throughout a branch-and-bound tree. Upper-bound ub_j^k is ub_j relative to node k and is calculated with regard to the decisions taken at k and its predecessor nodes.

Upper-bound ub_0

At each node k, let npv_i^k be the maximum net present value of stand i over the periods when i can still be harvested, that is,

$$\mathsf{npv}_i^k = \max_{(i,t)\in\mathcal{S}_k} \mathsf{npv}_{it},$$

where npv_{it} is the net present value from stand i if it is harvested in period t.

Let $\mathsf{npv}_{\mathrm{tot}}(x^k)$ be the sum of npv_i^k over set V. Upper-bound ub_0^k is given by

$$ub_0^k = \mathsf{npv}_{\mathrm{tot}}(x^k) + \mathrm{fnpv}(x^k).$$

Values $\mathsf{npv}_{\mathrm{tot}}(x^{k+1})$ and $\mathsf{npv}_{\mathrm{tot}}(x^{k+2})$ are calculated from $\mathsf{npv}_{\mathrm{tot}}(x^k)$ as follows:

- for the left branch, $\mathsf{npv}_{\mathrm{tot}}(x^{k+1}) = \mathsf{npv}_{\mathrm{tot}}(x^k) - \mathsf{npv}_{i_k t_k}$. If an element (i, t) such that stand i has the largest net present value in period t is also removed from $\mathcal{S}_k$, the difference between npv_{it} and the second largest net present value of i is also subtracted;
- for the right branch, $\mathsf{npv}_{\mathrm{tot}}(x^{k+2})$ is equal to the value of $\mathsf{npv}_{\mathrm{tot}}(x^k)$ minus the difference between $\mathsf{npv}_{i_k t_k}$ and the second largest net present value of stand i_k.

Upper-bounds ub_1, ub_2, ub_3 and ub_4

These bounds are calculated based on relaxations of P_0, *i.e.*on problems obtained from P_0 by eliminating or weakening some of its constraints.

The first relaxation, denoted by P_1, is defined by constraints (4) preventing two or more interventions in each stand in the horizon, constraints (5) and (6) limiting the volume of timber harvested in each period, ending-age constraint (7), maximum clearcut area and the greenup constraints (8), and binary constraints (12). Since set $\mathcal{H}'_t$ can be extremely large, constraints (9), (10) and (13) involving habitat variables are removed. Constraints (10), which impose a minimum in the total habitat area in each period, are replaced by constraints (14), that simply ensure, in each period, a minimum in the mature area.

$$M_t - \sum_{u=max\{1,t-(\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}}-1)\}}^{t} \sum_{c \in \mathcal{C}_u} \mathsf{s}_c z_{cu} \geq \mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}, \ \forall t \in \mathcal{T}, \tag{14}$$

where M_t is the area of all mature stands in period t.

Problem P_2 is similar to P_1 except that it does not contain volume constraints (5) and (6). Problems P_3 and P_4 are the linear relaxations of P_1 and P_2 respectively,*i.e.*the binary requirement on the variables Z_{ct} are replaced by nonnegativity constraints.

Upper-bounds $ub_1, \ldots, ub_4$ are, whenever is possible, the optimal solution values of $P_1, \ldots, P_4$, respectively. Since P_1 and P_2 are integer programs, they may not always be solved to optimality. In this case ub_1 and ub_2 are upper bounds on the optimal values of those problems. The above relaxation and upper bound definitions are summarized in Table 1.

Upper-bound	Problem (formulation)	Value of the upper-bound
ub_1	P_1 (objective (3), constraints (4)-(8), (12), (14)	optimal value or upper-bound
ub_2	P_2 (P_1 without volume constraints)	optimal value or upper-bound
ub_3	P_3 (linear relaxation of P_1)	optimal value
ub_4	P_4 (linear relaxation of P_2)	optimal value

Table 1: Upper-bounds $ub_j,\ j = 1, ..., 4$.

Due to the branching decisions, some extra constraints can be imposed on the problems associated to the nodes. We denote by P_j^k the problem corresponding to P_j at node k of the tree.

Let $\mathcal{L}_t^k$ be the set of all clearcuts in period t and $\mathcal{J}_k$ the set of all pairs (i, t) such that stand i is harvested in t ($i \in c$ for some $c \in \mathcal{L}_t^k$). Let $\mathcal{N}_k$ be the set of all pairs (i, t) such that i is not harvested in t ($\mathcal{N}_k = \mathcal{S}_0 \backslash (\mathcal{S}_k \cup \mathcal{J}_k)$). The following constraints are introduced, corresponding to the decisions already taken up to node k:

$$\sum_{c \in \mathcal{C}_t:\ c \cap c' = c'} z_{ct} = 1,\ \forall c' \in \mathcal{L}_t^k, \forall t \in \mathcal{T} \tag{15}$$

$$\sum_{c \in \mathcal{C}_t:\ i \in c} z_{ct} = 0,\ \forall (i, t) \in \mathcal{N}_k. \tag{16}$$

Constraints (15) impose that for each clearcut c' in each period, just one potential clearcut containing c' (or simply c') must be harvested in the period. Constraints (16) impose that for each stand that is not harvested in a certain period, no potential clearcut containing the stand is harvested in the period.

Problems P_j^{k+1} and P_j^{k+2} are obtained from P_j^k in the following way:

- for the left branch, P_j^{k+1}:
  - if stand i_k is added to some clearcut $c'' \in \mathcal{L}_{t_k}^k$, constraint (15) for c'' must be replaced by the one corresponding to the new clearcut. Otherwise, constraint (15) for $c' = \{i_k\}$ must be added; $\mathcal{L}_{t_k}^{k+1}$ is obtained from $\mathcal{L}_{t_k}^k$ according to these changes;
  - adding constraint (16) for each element removed from $\mathcal{S}_k$ except (i_k, t_k);
  - $\mathcal{J}_{k+1} = \mathcal{J}_k \cup \{(i_k, t_k)\}$;
  - $\mathcal{N}_{k+1} = \mathcal{N}_k \cup (\mathcal{S}_k \backslash (\mathcal{S}_{k+1} \cup \mathcal{J}_{k+1}))$;
- for the right branch, P_j^{k+2}:

– $\mathcal{L}_t^{k+1} = \mathcal{L}_t^k, \forall t$;
– adding constraint (16) for (i_k, t_k);
– $\mathcal{J}_{k+2} = \mathcal{J}_k$;
– $\mathcal{N}_{k+2} = \mathcal{N}_k \cup (\mathcal{S}_k \backslash (\mathcal{S}_{k+2} \cup \mathcal{J}_{k+2})) = \mathcal{N}_k \cup \{(i_k, t_k)\}$.

Observe that upper-bounds ub_j^{k+1} and ub_j^{k+2} are only needed to calculate when the solution corresponding to ub_j^k does not satisfy some new constraint.

4.2 Branch-and-bound implementation

Three different strategies to guide the search on the tree were implemented: depth-first, best-first with diving and beam search.

In depth-first search (DFS), the search is implemented though a last-in-first-out ($LIFO$) process on queue Q. The right branch solution is inserted into Q first, followed by the left branch solution. Therefore, the search descends on the left side of the tree until a leaf is reached, the so-called *diving* (the fastest way to reach a leaf).

In best-first search with diving (BFSD), the search descends on the left side of the tree until a leaf is reached. When a leaf is found, the search restarts at the Q element with the highest upper-bound and descends once again on the left side of the tree until a new leaf is reached. If there is more than one Q element with the same highest upper-bound, the last element inserted into Q is chosen.

In breadth-first search, all the solutions at the same level are searched for before exploring the next level. In beam search (BS), breadth-first search is parameterized, by limiting the number of solutions to branch per level. At each level, the generated partial solutions are sorted in ascending order of the upper-bound, and the β last solutions are branched while the other solutions are pruned. This strategy reduces both time and memory requirements relatively to breadth-first search, but does not guarantee finding the optimal solution.

Applying the first two strategies, the whole tree has been explored when Q is empty. In this case, the best solution found is an optimal solution. Branch-and-bound is used as a heuristic when the tree is partially explored.

5 Computational experiment

5.1 Instances

We used instances available at the web site `http://www.unbf.ca/fmos/`: *El Dorado*, a region in the EL Dorado National Forest in northern California (referred to in Goycoolea et al., 2005); *Stafford* a forest in British Columbia, in Canada; *Kittaning*4, *Bear Town*, *Phyllis Leeper*, and *Five Points*, forests in Pennsylvania;

WLC, *FLG*9 and *FLG*10. Only instances that have all required information available were used.

The number of stands ranges from 32 (Kittaning4) to 1363 (El Dorado) and the number of edges from 48 to 4087 (considering the weak adjacency). The length of the temporal horizon ranges from 3 to 9 periods. Some instances are tested with different number of periods T and the notation used is *instance*$_T$. Tables 2 and 3 summarize the characteristics of the instances.

Instance	No. stands	Weak adjacency No. edges	Strong adjacency No. edges	Strong adjacency No. cliques	No. periods	No. years per period
El Dorado	1363	4087	3617	2041	3	10
Stafford	1008	2113	2066	1163	3	10
$\text{FLG9}_{3/9}$	850	2524	2388	1420	3 / 9	5
$\text{FLG10}_{3/9}$	763	2262	2137	1269	3 / 9	5
Five $\text{Points}_{3/5}$	90	164	149	88	3 / 5	10
$\text{PhyllisLeeper}_{3/5}$	89	161	131	86	3 / 5	10
$\text{WLC}_{3/7}$	73	114	98	63	3 / 7	5
Bear $\text{Town}_{3/5}$	71	148	101	64	3 / 5	10
$\text{Kittaning4}_{3/5}$	32	48	47	25	3 / 5	10

Table 2: Size of the instances.

In order to use these instances several parameters had to be defined. Parameter α, the maximum deviation of the harvested volume in each period from the target volume, is set to 0.15 and parameter v_0, the target volume for each period, is calculated as the timber volume of all stands in the first period divided by the total number of periods, that is, $\mathsf{v}_0 = \sum_{i=1}^{n} \mathsf{v}_{i1}/T$ (*e.g.*Yoshimoto and Brodie, 1994). In order to obtain a feasible solution to problem P_0, the denominator of v_0 is replaced by $T+1$ for Bear Town_5, WLC_3 and PhyllisLeeper_5, $T+2$ for Bear Town_3, PhyllisLeeper_3, Stafford and El Dorado, and $T+3$ for FLG9_3 and FLG10_3. The minimum average age for the forest at the end of the planning horizon $\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$ is set to $(T+1)-T/2$. The minimum number of green-up periods G is set to 1, that is, adjacent potential clearcuts can simply not be harvested in the same period. The minimum habitat area in each period $H^{\min}_{\text{tot}}$ is set to 10% of the total area F. In the path between two habitats, the area of an intermediate mature patch (step or corridor) cannot be less than 5% of the minimum habitat area $\mathsf{H}^{\mathsf{min}}$. The coefficient of dispersion C in Equation (1) is set to 0.0001386, which was calculated considering the indicator $p_{lm} = 0.5$ for the distance d_{lm} of 5000 m. The minimum values for the connectivity index (I) were selected in order to be restrictive for each instance but not too much. The values range according to the instances and are displayed in Table 3. This table also shows the values of parameters $\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$, $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$, $A^{\max}$, $\mathsf{H}^{\mathsf{min}}$ and F. Further information about the instances is also displayed: $a\bar{g}e_1$ is the average age of the stands in the first period; $\bar{s}$ is the average area of the stands.

Parameter β, specifying the number of solutions to keep in each level on the tree of BS strategy is set to 5 for El Dorado, Stafford, FLG9 and FLG10 (larger instances), and for the other instances it is set to 100.

Instance	$a\bar{g}e_1$ (years)	$\mathsf{Age}_{\mathsf{cut}}^{\mathsf{min}}$ (years)	$\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}}$ (years)	$\bar{s}$ (ha)	$A^{\max}$ (ha)	$\mathsf{H}^{\mathsf{min}}$ (ha)	F (ha)	I
El Dorado	105.86	60	80	15.52	40	40	21147	0.2
Stafford	50.83	60	60	10.36	40	40	10444	0.005
$\mathrm{FLG9}_{3/9}$	31.91	40	40	11.76	46	46	10000	0.017 / 0.02
$\mathrm{FLG10}_{3/9}$	27.02	40	40	13.10	46	46	10000	0.005 / 0.02
Five $\mathrm{Points}_{3/5}$	63.11	60	60	7.52	40	40	677	0.08 / 0.08
Phyllis $\mathrm{Leeper}_{3/5}$	94.38	80	80	7.26	40	40	646	0.03 / 0.03
$\mathrm{WLC}_{3/7}$	46.58	40	40	12.28	40	40	897	0.017 / 0.039
Bear $\mathrm{Town}_{3/5}$	95.49	80	80	7.69	40	40	546	0.1 / 0.03
$\mathrm{Kittaning4}_{3/5}$	66.59	60	60	7.44	40	40	238	0.05 / 0.05

Table 3: Values of $a\bar{g}e_1$ (average age of the stands in the first period), $\bar{s}$ (average area of the stands), F (area of the forest) and parameters $\mathsf{Age}_{\mathsf{cut}}^{\mathsf{min}}$ (minimum harvest age), $\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}}$ (minimum mature age), $A^{\max}$ (maximum clearcut area), $\mathsf{H}^{\mathsf{min}}$ (minimum habitat area) and I (minimum value for the connectivity index in each period).

For computing the distance between two patches, we consider the distance between two stands (represented as polygons) as the minimum Euclidean distance between their vertices. The edge-to-edge distance between two mature patches is approximated by the minimum distance between their stands and is set to zero when both patches share at least one point.

5.2 Results and discussion

The computational experiment was executed on a desktop computer with an Intel Core 2 processor running at 2 GHz, with 2 GB of RAM. Branch-and-bound was implemented in the Python language. Cplex 12.2 was used to solve problems P_1, P_2, P_3 and P_4, giving respectively the upper-bounds ub_1, ub_2, ub_3 and ub_4 throughout the branch-and-bound trees. All strategies were allowed to run for two hours at the most.

Let bub denote the best upper-bound, *i.e.*the minimum value of upper bounds $ub_j, j > 0$ obtained with DFS or BFSD strategies over all unexplored nodes. BS is not considered for bub because trees are partially explored. The quality of the best solution found x^* is measured by using the deviation (in percentage) of $\mathrm{fnpv}(x^*)$ from bub, $gap = (bub - \mathrm{fnpv}(x^*)) / \mathrm{fnpv}(x^*) \times 100$. Tables 4 and 5 show the CPU time and the gap of x^* for all strategies.

	DFS									
	ub_0		ub_1		ub_2		ub_3		ub_4	
Instance	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)
El Dorado	2625	**19.56**	2896	19.61	2376	19.61	2366	19.61	2921	19.61
Stafford	5847	**0.56**	6031	**0.56**	5539	**0.56**	5162	**0.56**	5325	**0.56**
$FLG9_3$	7023	9.27	7144	8.87	657	9.40	5509	9.2	539	9.4
$FLG9_9$	1807	8.88	882	**8.8**	862	**8.8**	6606	**8.8**	812	**8.8**
$FLG10_3$	6191	8.84	6020	8.42	5233	8.85	4337	8.85	4470	8.85
$FLG10_9$	935	7.97	897	**7.43**	843	**7.43**	755	**7.43**	755	**7.43**
Five $Points_3$	308	0.31	248	0.89	1.1	4.29	1.1	4.29	1.1	4.29
Five $Points_5$	734	1.43	447	0.55	1.44	1.61	1.39	1.61	1.5	1.61
Phyllis $Leeper_3$	6827	2.72	6963	3.44	1.14	3.5	1.18	3.5	1.11	3.5
Phyllis $Leeper_5$	65.8	**1.26**	3.50	1.46	3.65	1.46	3.25	1.46	3.54	1.46
WLC_3	3092	0.63	7159	0.49	0.43	1.53	6443	0.87	0.44	1.53
WLC_7	185	**0.57**	1138	**0.57**	0.89	1.05	0.89	1.05	0.90	1.05
Bear $Town_3$	6527	0.28	2177	**0.26**	1673	0.29	4779	0.29	1395	0.29
Bear $Town_5$	6270	0.58	5326	**0.52**	3282	0.97	6749	0.73	1601	0.97
$Kittaning4_3$	59.01	**0**	634	**0**	15.39	6.3	13.58	6.3	10.77	6.3
$Kittaning4_5$	15.58	**0**	95.73	**0**	24.09	10.1	5153	8.77	5519	9.83

	BFSD									
	ub_0		ub_1		ub_2		ub_3		ub_4	
Instance	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)
El Dorado	-	-	2448	19.61	2545	19.61	2512	19.61	2389	19.61
Stafford	-	-	6133	**0.56**	6487	**0.56**	5262	**0.56**	5489	**0.56**
$FLG9_3$	-	-	145	9.53	166	9.53	146	9.53	148	9.53
$FLG9_9$	(O)	(O)	861	**8.8**	867	**8.8**	847	**8.8**	767	**8.8**
$FLG10_3$	-	-	234	8.91	249	8.91	234	8.91	225	8.91
$FLG10_9$	-	-	917	**7.43**	742	**7.43**	744	**7.43**	753	**7.43**
Five $Points_3$	(O)	(O)	130	**0**	0.99	4.29	1.1	4.29	1	4.29
Five $Points_5$	(O)	(O)	1347	**0**	1.36	1.61	-	-	1.39	1.61
Phyllis $Leeper_3$	(O)	(O)	(O)	(O)	1.07	3.5	6484	1.02	1.07	3.5
Phyllis $Leeper_5$	3.64	1.46	3.78	1.46	3.64	1.46	3.84	1.46	3.99	1.46
WLC_3	(O)	(O)	0.41	1.53	5398	0.84	0.42	1.53	6899	0.78
WLC_7	(O)	(O)	0.88	1.05	0.84	1.05	0.83	1.05	0.83	1.05
Bear $Town_3$	(O)	(O)	(O)	(O)	3.23	0.67	1933	0.27	2.16	0.67
Bear $Town_5$	(O)	(O)	(O)	(O)	4.09	1.18	4564	0.59	3.14	1.18
$Kittaning4_3$	(O)	(O)	40.40	**0**	362	**0**	78.64	**0**	290	**0**
$Kittaning4_5$	(O)	(O)	77.95	**0**	77.95	**0**	147	**0**	2340	**0**

Table 4: Computational results with DFS and BFSD strategies for all upper-bounds (branch-and-bound time and gap of the best solution found (in percentage)). (O) - out of memory.

	BS									
	ub_0		ub_1		ub_2		ub_3		ub_4	
Instance	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)	time (s.)	gap (%)
El Dorado	-	-	-	-	-	-	-	-	-	-
Stafford	-	-	-	-	-	-	-	-	-	-
$FLG9_3$	4459	9.03	-	-	5888	**8.4**	3599	10.81	5007	8.67
$FLG9_9$	-	-	(O)	(O)	-	-	-	-	-	-
$FLG10_3$	4545	8.4	-	-	5333	7.31	3871	**5.23**	4059	7.06
$FLG10_9$	-	-	(O)	(O)	-	-	-	-	-	-
Five $Points_3$	188	4.53	3376	**0**	-	-	2368	13.3	2976	2.25
Five $Points_5$	521	81.53	6604	**0**	6174	2.79	-	-	-	-
Phyllis $Leeper_3$	333	3.79	-	-	-	-	3783	**0.12**	-	-
Phyllis $Leeper_5$	562	3.25	-	-	-	-	-	-	-	-
WLC_3	-	-	-	-	-	-	317	**0.098**	-	-
WLC_7	306	4.33	(O)	(O)	-	-	1432	5.94	-	-
Bear $Town_3$	127	1.57	-	-	-	-	2322	0.53	-	-
Bear $Town_5$	301	2.75	(O)	(O)	-	-	5598	0.96	-	-
$Kittaning4_3$	10.41	1.23	295	**0**	227	**0**	131	**0**	138	**0**
$Kittaning4_5$	315	20.40	282	**0**	315	2.5	114	0.02	202	5.6

Table 5: Computational results with BS strategy for all upper-bounds (branch-and-bound time and gap of the best solution found (in percentage)). (O) - out of memory.

The computer ran out of memory with strategy BFSD and upper-bound ub_0 for Kittaning4, Bear Town, WLC, Phyllis $Leeper_3$, Five Points and $FLG9_9$, with BFSD and ub_1 for Bear Town and Phyllis $Leeper_3$, and with BS and ub_1 for Bear $Town_5$, WLC_7, $FLG9_9$ and $FLG10_9$. Not always was branch-and-bound able to find a feasible solution with BFSD and with BS.

Branch-and-bound was able to find optimal or approximate solutions for all instances in reasonable time. For Kittaning4 and Five Points optimal solutions are found. All strategies gave the optimum for Kittaning4. The optimum for Five Points was found with BFSD and BS strategies with upper bound ub_1, but BFSD found the solution earlier. For Bear Town, WLC, Phyllis $Leeper_3$ and Stafford, branch-and-bound was able to find a solution under 1% of the optimum. DFS/ub_1 provided the best solution for Bear Town, but all strategies gave solutions under 1% of the optimum. BS with ub_3 produced the best solution for WLC_3 and Phyllis $Leeper_3$, and DFS with upper-bounds ub_0 and ub_1 provided the best solution for WLC_7. For Phyllis $Leeper_5$, $FLG9_9$ and $FLG10_9$, Stafford and El Dorado no DFS/ub_j or BFSD/ub_j were performed significantly better. BS/ub_2 and BS/ub_3 produced the best solution for $FLG9_3$ and $FLG10_3$, respectively.

For DFS and BFSD, ub_1 gave more lower gaps than the other upper bounds. For BS, were ub_1 and ub_3. Although addressing volume constraints generally increases the difficulty of solving the integer programming formulations (*e.g.*Vielma et al., 2007; Constantino et al., 2008), they seem to improve the performance of branch-and-bound (giving tighter upper bounds). Additionally, solving linear relaxations (P_3 or P_4) or the integer formulations (P_1 or P_2) for larger instances, the former to optimality and the second partially, did not lead to results significantly different. The linear relaxations might give weaker upper bounds but are solved in less time. For the large instances, DFS ansd BFSD with ub_1 were very close concerning the gaps, but BFSD spent less time obtaining the best solution found. BS seems to be inferior to the others strategies.

The upper-bounds ub_j with $j > 0$ for DFS and BFSD were only calculated after the first feasible solution had been obtained, in order to save time. This is not undertaken with BS, because the criterion used to select a certain number of nodes at each tree level is the upper bound value. Probably this is why BS did not give a feasible solution for large instances within the allowed CPU time. And also it might explain the number of non-pruned nodes (Figure 2), that was usually much larger with DFS and BFSD. The number of non-pruned nodes was usually much larger with ub_3 and ub_4 than with ub_1 and ub_2, respectively. Usually, ub_0 gave large numbers of non-pruned nodes. For BFSD/ub_0, queue Q was often very large and the computer ran out of memory.

In order to evaluate the effect of the habitat availability constraints on the net present values, we made some computational tests (the results are presented in Table 6). As would be expected, we observed that for almost all instances it was not possible to include the habitat availability without reducing the attainment of the net present value of timber harvested. In general, the impact of the constraints on the connectivity index is greater than that of the habitat area constraints. The reduction in the net present value with the inclusion of habitat availability was relatively small for the majority of the instances. Only for the instance FLG9_9, branch-and-bound found a solution with a better net present value. One explanation for the relatively low cost of addressing the habitat availability is that the definition of mature forests (older than 40, 60 or 80 years old) gives a good supply of mature patches over time, providing space for alternative solutions.

To evaluate the effect of the connectivity index on the spatial arrangement of the habitats, we compared solutions obtained by relaxing the constraints on the connectivity index ($\mathsf{I} = 0$) with solutions obtained with the largest possible value for I (in terms of problem feasibility). Table 7 displays, for the large instances and both values of I, the number and the total area of habitats and the average distance between one pair of habitats, observed for the last period, with DFS strategy and upper-bound ub_1. The average distance is computed as the sum of the distances between all pairs of habitats divided by the number of these pairs. A map representation of the solution obtained for the instance Five Points$_3$ with DFS/ub_1 is shown in Figure 3.

Figure 2: Non-pruned nodes of the branch-and-bound tree for all strategies. Instances running out of memory are not displayed.

Instance	$\mathsf{I}=0$, $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}=0$ fnpv	$\mathsf{I}=0$, $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}>0$ fnpv (%)	$\mathsf{I}>0$, $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}=0$ fnpv (%)	$\mathsf{I}>0$, $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}>0$ fnpv (%)
El Dorado	2047519.8	0	- 1.12	-1.12
Stafford	137896523.0	-0.1	-0.014	-0.014
FLG9_3	45536003.1	-0.02	-0.20	-0.23
FLG9_9	56454241.9	0	+0.07	+0.07
FLG10_3	42689742.5	-0.02	-0.02	-0.04
FLG10_9	53252414.8	0	-0.03	-0.03
Five Points_3	544257.6	0	0	0
Five Points_5	423376.2	0	0	0
Phyllis Leeper_3	5588674.2	-0.001	-1.42	-1.49
Phyllis Leeper_5	5871743.1	+0.12	-0.18	-0.18
WLC_3	4641062.5	-0.0003	-0.04	-0.005
WLC_7	4202538.1	-0.03	-0.29	-0.39
Bear Town_3	4540642.1	-0.002	-0.013	-0.013
Bear Town_5	4672405.5	+0.0002	0	0
Kittaning4_3	1627147.6	0	-1.24	- 1.36
Kittaning4_5	1542770.7	0	-0.25	-1.97

Table 6: The reductions (-) or gains (+) on the net present values (in percentage) whit the inclusion of habitat availability (the minimum value for the connectivity index in each period $\mathsf{I} > 0$ is presented in Table 3; the minimum total habitat area in each period $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}} > 0$ is set to 10% of F) with DFS strategy and upper-bound ub_1.

Instance	$\mathsf{I}=0$			Largest possible value for I			
	No. habitats	Habitat area (ha)	Average distance (m)	I	No. habitats	Habitat area (ha)	Average distance (m)
El Dorado	13	9955.17	518.7	0.25	8	11244.89	500.79
Stafford	17	1044.58	2605.82	0.006	25	5794.4	497.84
FLG9_3	14	1478	1229.73	0.03	19	2464	1015.33
FLG9_9	17	2783	541.88	0.036	15	3159	509.83
FLG10_3	15	1069	2118.58	0.017	19	1944	1534.22
FLG10_9	19	2814	747.87	0.046	11	3587	548.65

Table 7: Number of habitats, total area of the habitats and the average distance between one pair of habitats, in the last period, obtained with $\mathsf{I} = 0$ and the largest possible value for I (in terms of problem feasibility), using DFS strategy and upper-bound ub_1.

The connectivity index enhanced not only the connectivity between habitats (by reducing the inter-habitat distances) but also the area of all habitats (by increasing the number of habitats or the area of each habitat). In other words, this index seems to be a very good measure for the habitat availability constraints.

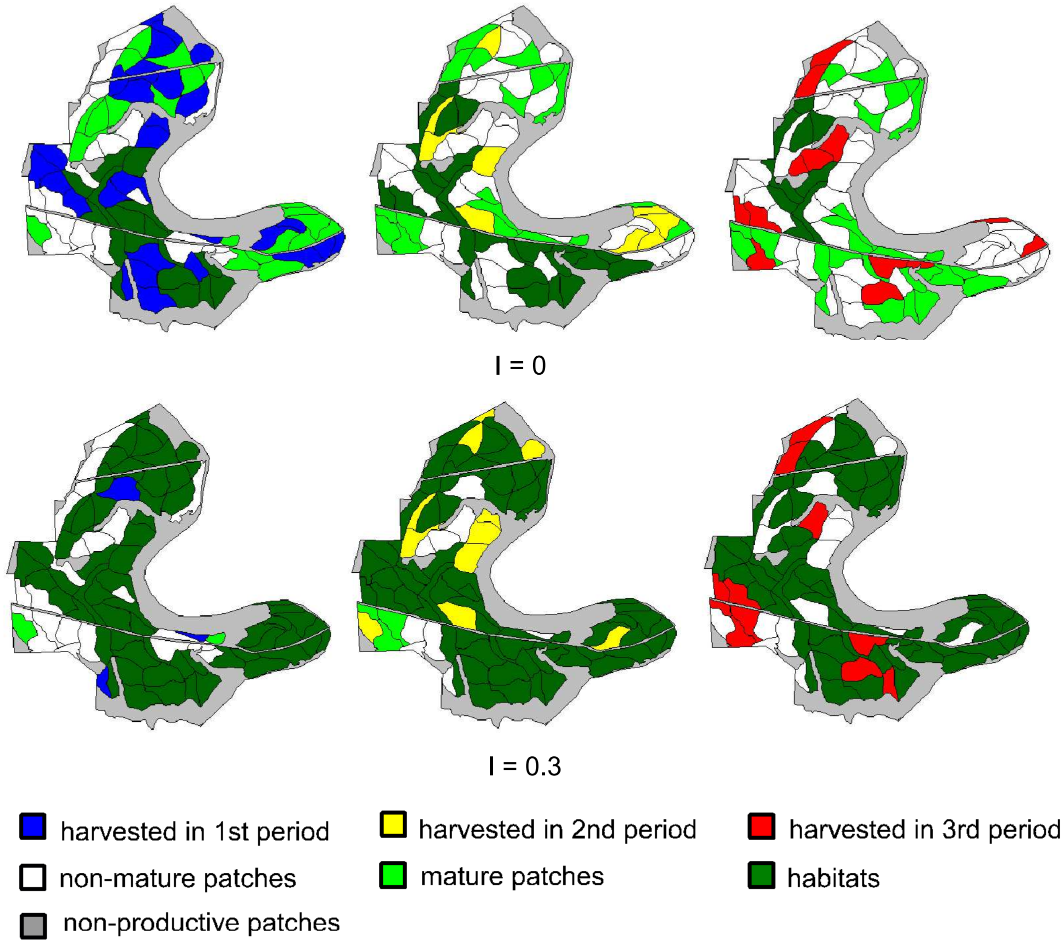

Figure 3: Map representation of the solution obtained for Five Points$_3$ with DFS and ub_1.

6 Conclusions

This study presents a branch-and-bound method designed specifically for forest harvest scheduling problems addressing habitat availability concerns. Two main spatial issues affect habitat availability: the area of all habitats and the connectivity between them. Recent research on these problems has just addressed the area of all habitats. In this work, we propose to handle both. We use constraints on the so-called probability of connectivity index, a non-linear measure, to increase the inter-habitat connectivity. Three different strategies are tested to guide the search on the branch and bound tree, depth-first search, best-first search with diving and beam search. Five different upper-bounds are considered to prune a node, four of which are obtained by solving harvest scheduling problems with no habitat availability constraints. An integer programming model (the so-called cluster formulation) and relaxations are used for these problems.

Branch-and-bound was tested with forests ranging from 32 to 1363 stands, and temporal horizons ranging from three to nine periods were employed. The main objective of the computational tests is to assess the ability of branch-and-bound to obtain solutions of a certain quality in a reasonable time (up to two hours). Another objective is to determine which strategies and upper-bounds are better. We also studied the impact of some values of the connectivity index on the net present value and on the spatial arrangement of the habitats.

The results show that branch-and-bound was able to solve optimally or approximately the problems for all instances in a reasonable time. For four instances, branch-and-bound found optimal solutions, and for other six instances it provided solutions within 1% of the optimum. For most of the instances ranging from 763 to 1363 stands, the solutions obtained stood within 10% of the optimal solution. Depth-first search and best-first search with diving seemed to be better suited for larger instances, although the second strategy spent less time obtaining the best solution found. In general, beam search appeared to be inferior to these two strategies. The best upper-bounds were obtained by solving the integer programming model that includes volume constraints or its linear relaxation.

For the instances tested, habitat availability increasing was obtained at the expense of small reductions in the net present value. The definition of mature forests giving a good supply of mature patches over time might be one explanation for this tendency. Constraints on the connectivity index may enhance themselves both the connectivity between patches and the total habitat area, that is the habitat availability.

The computational results of this work are exploratory and it seems clear that more research is required to fully understand the implications of the inclusion of habitat availability constraints in the models. Further research in computationally effective ways of solving harvest scheduling problems with constraints on habitat availability is necessary, and branch-and-bound may cast some light on successful approaches.

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Chapter 4

Forest harvest scheduling with clearcut and core area constraints

This chapter presents the second paper which was published in the Annals of Operations Research [Neto et al., 2016].

Forest harvest scheduling with clearcut and core area constraints

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Abstract

Many studies regarding environmental concerns in forest harvest scheduling problems deal with constraints on the maximum clearcut size. However, these constraints tend to disperse harvests across the forest and thus to generate a more fragmented landscape. When a forest is fragmented, the amount of edge increases at the expense of the core area. Highly fragmented forests can neither provide the food, cover, nor the reproduction needs of core-dependent species. This study presents a branch-and-bound procedure designed to find good feasible solutions, in a reasonable time, for forest harvest scheduling problems with constraints on maximum clearcut size and minimum core habitat area. The core area is measured by applying the concept of subregions. In each branch of the branch-and-bound tree, a partial solution leads to two children nodes, corresponding to the cases of harvesting or not a given stand in a given period. Pruning is based on constraint violations or unreachable objective values. The approach was tested with forests ranging from some dozens to more than a thousand stands. In general, branch-and-bound was able to quickly find optimal or good solutions, even for medium/large instances.

Keywords: Forest planning, Core area, Edge effect, Clearcut, Integer programming, Branch-and-bound.

1 Introduction

Environmental concerns, such as protecting wildlife, reducing erosion, and preserving scenic beauty, have led to important modifications in forest harvest scheduling problems. A common practice in many countries to address these concerns has been to restrict the areas of clearcuts, particularly in old-growth (mature) forests (for a comparison between forest policies around the world, we refer those interested to McDermott et al. (2010)). Nevertheless, this policy also causes a dispersion of a greater number of smaller clearcuts across the forest, and thus a more fragmented forest (Franklin and Forman, 1987). Forest fragmentation occurs with the segmentation of a large and continuous tract of mature forest to smaller patches, separated from each other by patches of early successional species, roads, agriculture, urbanization or other development. This process disturbs the habitat of many animals and plants, not only because it reduces the area that is left as forest but also because affects other biophysical aspects of the forest, such as forest structure, temperature, moisture and light conditions.

Franklin and Forman (1987) classify a mature forest patch into edge and core area. *Core area* is defined as the interior area of the patch where ecological functioning is not impacted by the effect of immediate surrounding conditions, the so-called *edge effect*. The edge effect corresponds to a buffer area (edge), separating the core area from outside influences, and is due to clearcuts (or patches of early successional species) and non-forest patches. Some plant and animal species have adapted to forest edges (edge-dependent species), but many others are more dependent on forest interiors (core-dependent species). Edges are preferred by species that require both shelter and open browsing areas. Many forest-nesting birds shun edges because of the increased risk of predation or nest parasitism, as well as inadequate temperature, light and moisture conditions, or insufficient food. When a forest is fragmented, the amount of edge increases at the expense of core area.

The core area of a forest patch is determined by the area, shape and immediate surrounding conditions of the patch (Franklin and Forman, 1987; Baskent and Jordan, 1995). Small mature patches have a proportionately high amount of edge. As an example (Figure 1), consider that a mature patch must be at least 100 metres from the boundary before it can be considered a core area. Thus, a $200\,\mathrm{m} \times 200\,\mathrm{m}$ square mature patch (four hectares) or a three hectares round mature patch contain no interior habitat (100% edge), and would be unlikely to support interior wildlife species. A circular mature patch would have to be almost eight hectares in size to contain just one hectare of interior habitat (87.5% edge approximately). On the other hand, large mature patches do not necessarily provide a greater opportunity for core-dependent species if their shape is too elongated or complex. Mature patches with roughly circular shape have proportionally much more core area, and thus are more favorable to core-dependent species than those similar in area but with elongated or complex shapes (Figure 2). Finally, the distance from a patch boundary inward to a point where edge effects are eliminated increases with increasing contrast between the patch and surrounding conditions.

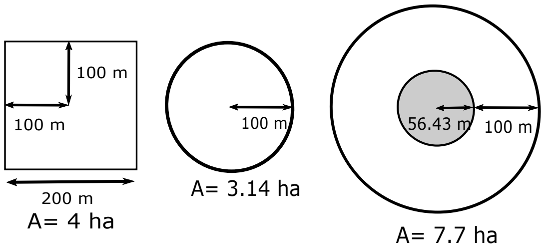

Figure 1: Core area (shaded area) and edge (white area) for three different patch sizes.

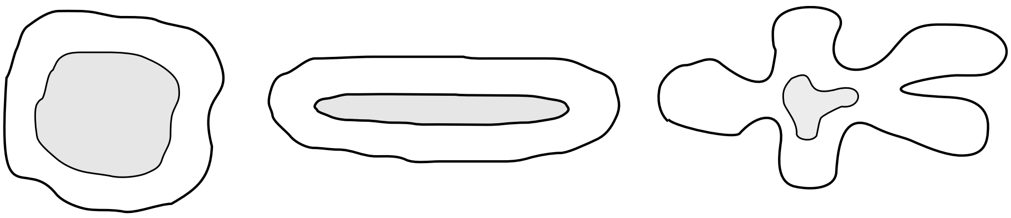

Figure 2: Core area (shaded area) and edge (white area) for three different patch shapes.

Forest fragmentation leads not only to shrinking of the core habitat area but also to the weakness of the inter-habitat connections (Harris, 1984; Kurttila et al., 2002). As a forest becomes fragmented, mature patches become separated from one another by relatively inhospitable terrain. However, wildlife should be able to move freely from one forest patch to another. This movement allows for interbreeding, creating genetically stronger populations and ensuring that suitable habitats can be filled. Large distances between mature patches may prevent this movement and are an impediment for migrating wildlife (wildlife attempting to cross between patches becomes temporarily vulnerable to predators, for example). For this reason, corridors between isolated patches can help wildlife by providing routes through which they can travel. Corridors also benefit plants, facilitating seed dispersal and subsequent establishment of plants in new areas.

Integrating forest fragmentation into forest harvest scheduling problems adds substantial complexity to the models and solution techniques. Several studies involving this integration have been given. Hof et al. (1994) indirectly modeled habitat fragmentation using mixed integer linear programming formulations that focus on wildlife growth and dispersion as a dynamic and a probabilistic process. Martins et al. (1999); Rebain and McDill (2003a,b); Martins et al. (2005) constrain the minimum total area of mature patches with a minimum area requirement. Tóth et al. (2006) constrain or minimize the total perimeter of the mature patches. All five studies endeavored to solve mixed integer linear programming models, also with constraints on the maximum clearcut area. The approach described in Öhman and Wikström (2008) relies in multi-objective programming, where the total perimeter of the mature patches is regarded as an additional objective besides the net present value. Constraints on the clearcut area are not considered in that study. In order to circumvent computational limitations to the use of exact methods, some studies proposed heuristics to solve forest planning problems with both constraints on total area of mature patches with a minimum area requirement and clearcut area (simulated annealing and hybrid heuristics by Falcão and Borges (2002), tabu search by Caro et al. (2003)). Öhman and Lämås (2005) used the shape index, a ratio perimeter to area (McGarigal et al., 2002), as a criterion for decreasing the fragmentation of old forest. A two-objective problem was solved with simulated annealing, whose aim is to maximize the net present value and minimize the shape index for the forest. Constraints on the clearcut area are not considered in this study. Neto et al. (2013) presented a branch-and-bound approach to solve forest harvest scheduling problems with constraints on both the total area of mature patches with a minimum area requirement and inter-habitat connectivity. This study used the probability of connectivity index (Saura and Pascual-Hortal, 2007) to measure inter-habitat connections, and constraints on the clearcut area were included.

Core area can be considered in the models either directly or indirectly. Approaches that directly include the core area have been developed by Wei and Hoganson (2007) and Zhang et al. (2011), who used mixed integer programming formulations; Öhman and Eriksson (1998); Öhman (2000); Öhman et al. (2002) have proposed simulated annealing, and Hoganson et al. (2005); Wei and Hoganson (2008) used dynamic programming-based heuristics. None of these works considered constraints on the clearcut area. Indirect approaches use patch area and shape, which, together, determine the core area, and can be used to approximate the core area. However, there is no single indicator that summarizes all characteristics of shape. For example, shape index values range from one (perfect circle) to higher values (irregular shapes), but two patches of different shapes may share the same shape index value (Baskent and Jordan, 1995). There are many other approaches that try to capture the essential characteristics of shapes (Davis, 1977; Moellering and Rayner, 1981; Xia, 1996; Wentz, 2000). As far as we know, the only indirect approaches to address core area used in the literature are the shape index and measures of perimeter and area. The other approaches seem to be rather complex to be used within harvest scheduling models.

Addressing the issue of forest fragmentation into forest planning models with no constraints on clearcut area may not prevent excessive harvesting of a particular area. The objective of this study is to investigate the possibility of solving forest harvest scheduling problems with both constraints on the core area and clearcut area. Here the core area is directly measured using the concept of subregions (see, *e.g.*, Wei and Hoganson, 2007). We designed a branch-and-bound procedure to solve these problems, inspired in the approach of Neto et al. (2013). This technique can be used as an exact method or, if interrupted, as a heuristic. In the latter case, the algorithm is stopped when the solution is considered satisfactory, or a time limit is reached.

The outline of this paper is as follows. In sections 2 and 3, we describe and formulate the forest planning problem. In Section 4, we present the branch-and-bound procedure. In Section 5, we report on computational experiments, carried out on forests ranging from 32 to 1363 stands. In the last section, we present some conclusions.

2 Problem definition

The basic unit in forest inventory and planning is usually a stand compartment, which is in general an ecologically homogenous forest area (*e.g.* in species composition, age and condition). We use the term *cluster* to describe a set of contiguous homogeneous stands. A patch is a cluster distinct from its surroundings (in other words, a maximal cluster). A clearcut is a patch with no trees. A mature patch is a forest patch that is older than a certain age. For the sake of simplicity, we use the term *habitat* to describe a mature cluster with a minimum core area requirement. We assume that this type of cluster meets the life needs (*e.g.* food and shelter) of core-dependent species.

The harvest scheduling problem that we shall consider in this work consists of selecting, for each period in the planning horizon, a set of stands to be harvested, in order to maximize the timber's net present value. The stand selection is subject to several restrictions, spatial (depending on the relative arrangement of stands) and non-spatial.

As non-spatial constraints, we consider:

- **Volume constraints:** lower and upper bounds on the volume of timber harvested in each period.
- **Average ending age constraints:** a minimum average age for the forest at the end of the planning horizon.

Volume constraints ensure a regular production of timber. The average ending-age constraint helps to prevent the model from over-harvesting the forest.

As spatial constraints, we consider:

- **Maximum clearcut size constraints:** a maximum in the area of each clearcut. For the sake of simplicity, it is assumed that the green-up time (minimum number of periods in which stands adjacent to a clearcut cannot be harvested) is one period.

- **Total habitat area constraints:** a minimum in the total area of habitats (edge and core area) in each period. It is assumed that a habitat is a mature cluster meeting a minimum core area.
- **Total core area constraints:** a minimum in the total core area inside habitats in each period.

We shall refer to this problem as P. It is assumed that each stand is harvested at the most once in the planning horizon, *i.e.*, the minimum rotation in the stand is longer than the latter. It is also assumed that harvesting occurs at the beginning of the periods, and all periods have the same length. Minimum harvest age is considered, and a harvested stand may become mature within the planning horizon.

We assume that an area in the forest is considered core if it is composed by mature trees and is not impacted by the effect of clearcutting (edge effect). For the sake of simplicity, it is further assumed that this edge effect occurs only after the clearcutting and during the green-up time, which in this case corresponds to the period of the intervention. Moreover, we assume the edge effect induced by a harvested stand attains a region — the *impact zone* — starting in the stand's boundary up to a constant distance outward (Baskent and Jordan, 1995).

An example of a forest with three stands (Example 1) is used for the purpose of illustration. The impact zone of stand A is the region corresponding to $\{B_3, B_4, C_2, C_4\}$, which means that if A is harvested (and there are no interventions in stands B and C), $\{B_3, B_4\}$ and $\{C_2, C_4\}$ will be the edges in B and C, respectively. The width of the surrounding impact zone represents the depth of the edge effect. For some species $100\,\mathrm{m}$ depth may represent a reasonable distance to approximate edge conditions, while for other species this distance may be substantially smaller or larger.

Intersecting the impact zones of all stands and the stands themselves determines a set of *subregions*. Core area is measured using these subregions (Öhman and Eriksson, 1998; Wei and Hoganson, 2007, 2008; Zhang et al., 2011). Let $\mathcal{R}$ be the set of the subregions of a forest.

Example 1. *Figure 3 provides an example of a mature forest with three stands, A, B and C, before and after intersecting the stands and their impact zones. In this case,* $\mathcal{R} = \{A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4\}$.

Each subregion is included in one stand (*host stand*), and either (i) is in the intersection of the impact zones of a set of adjacent stands (which we call *invasive stands*), or (ii) is not in the impact zone of any stand, and thus is what we call an *inner region*. A subregion of type (i) becomes a piece of edge if the host stand is mature and at least one of the invasive stands is harvested. It is core area if the host stand is mature and there is no invasive stand that is harvested. Therefore, in this work, a core area may have adjacent non-harvested stands that are not mature; an inner region is core area whenever the host stand is mature.

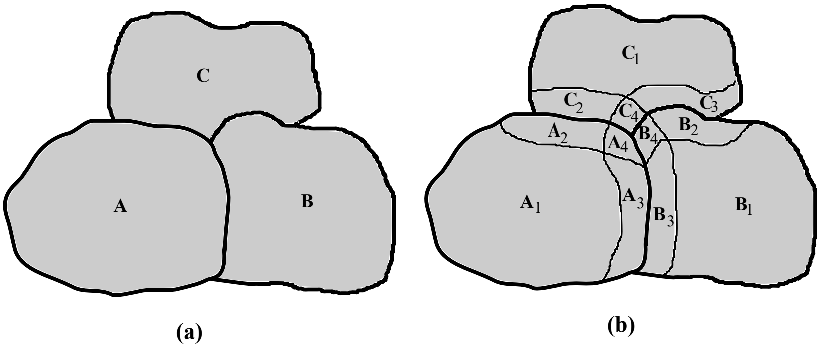

Figure 3: Forest considered in Example 1. (a) Three-stand mature forest. (b) Subregions after intersecting the stands and their impact zones. The impact zones of stands A, B and C are, respectively, $\{B_3, B_4, C_2, C_4\}$, $\{A_3, A_4, C_3, C_4\}$, $\{A_2, A_4, B_2, B_4\}$.

Example 1 (continued). *Table 1 displays all subregions for Figure 3, their host stands and, whenever they belong to an impact zone, their invasive stands (the area of each subregion is also shown, for further utilization). As an instance, A_4 is core area if the host stand (A) remains mature and the invasive stands (B and C) are not harvested, and becomes a piece of edge if B or C are harvested and A remains mature; C_1 is core area as long as the host stand (C) remains mature.*

Subregion	Type	Host stand	Invasive stands A	B	C	Area (ha)
A_1	(ii)	A				0.75
A_2	(i)	A			X	0.1
A_3	(i)	A		X		0.1
A_4	(i)	A		X	X	0.05
B_1	(ii)	B				0.75
B_2	(i)	B			X	0.1
B_3	(i)	B	X			0.1
B_4	(i)	B	X		X	0.05
C_1	(ii)	C				0.75
C_2	(i)	C	X			0.1
C_3	(i)	C		X		0.1
C_4	(i)	C	X	X		0.05

Table 1: Subregions obtained after intersecting the stands and the impact zones (X indicates an invasive stand).

Let $\mathcal{I}_r$ be the set of stands that determine whether subregion $r \in \mathcal{R}$ is core area. In case (i), $\mathcal{I}_r$ encompasses the host and the invasive stands. In case (ii), $\mathcal{I}_r$ is simply the host stand. Subregion r is core area when there is no stand in $\mathcal{I}_r$ that is harvested and the host stand is mature.

Example 1 (continued). *Subregions are determined by* $\mathcal{I}_{A_1} = \{A\}$, $\mathcal{I}_{A_2} = \{A, C\}$, $\mathcal{I}_{A_3} = \{A, B\}$, $\mathcal{I}_{A_4} = \{A, B, C\}$, $\mathcal{I}_{B_1} = \{B\}$, $\mathcal{I}_{B_2} = \{B, C\}$, $\mathcal{I}_{B_3} = \{B, A\}$, $\mathcal{I}_{B_4} = \{B, A, C\}$, $\mathcal{I}_{C_1} = \{C\}$, $\mathcal{I}_{C_2} = \{C, A\}$, $\mathcal{I}_{C_3} = \{C, B\}$, $\mathcal{I}_{C_4} = \{C, A, B\}$.

Consider the illustrations in Figure 4:

- *If only stand B is harvested, then all subregions $r \in \mathcal{R}$ for which $B \in \mathcal{I}_r$ are not core areas (in this case, subregions $A_3, A_4, B_1, B_2, B_3, B_4, C_3, C_4$). As the host stands A and C remain mature, the other subregions — A_1, A_2, C_1, C_2 — are core areas.*
- *If host stand A remains mature and both stands B and C are harvested, then the core area will be just A_1 (*i.e.*, the only subregion r such that $B, C \notin \mathcal{I}_r$).*

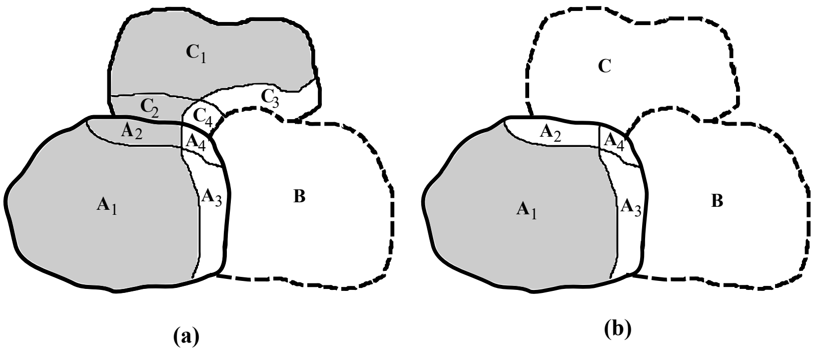

Figure 4: (a) After stand B is harvested A_1, A_2, C_1 and C_2 are core areas, and A_3, A_4, C_3 and C_4 are edges. (b) After stands B and C are harvested A_1 is core area and A_2, A_3 and A_4 are edges.

3 Model

This section presents an integer programming model for problem P. Three main basic integer programming models for the harvest scheduling problem with constraints on maximum clearcut area have been described in the literature. The path formulation (Martins et al., 1999; McDill et al., 2002; Murray and Weintraub, 2002; Crowe et al., 2003) encompasses an exponential number of constraints. The cluster formulation (Martins et al., 1999; McDill et al., 2002; Martins et al., 2005; Goycoolea et al., 2005; Vielma et al., 2007; Martins et al., 2012) encompasses an exponential number of variables. The bucket formulation (Constantino et al., 2008) has a polynomial number of variables and constraints. It has been proven theoretically that the LP bound of the cluster formulation is tighter than those of the other two formulations (Goycoolea et al., 2005; Martins et al., 2012). As the branch-and-bound described in this study uses LP bounds of the harvest scheduling problem with constraints on the maximum clearcut area, we propose to model P based on the cluster formulation.

The model selects clearcuts and habitats in order to maximize the timber's net present value such that spatial and non-spatial constraints are satisfied. All potential clearcuts and habitats are defined a priori. To identify these clusters, we must define adjacency between stands. From a point of view of aesthetics, rooted in beauty and visual appreciation, and also of other forest values as soil erosion and water quality, we consider to be reasonable to assume that a pair of harvested clusters sharing only one single point are non-adjacent and then belong to two different clearcuts. From a point of view of protecting wildlife, regarding the flow of genetic material throughout landscape, we consider to be reasonable to assume that a pair of mature clusters sharing only one single point are adjacent and then belong to the same patch. In this case, total habitat area constraints and total core area constraints are less restrictive than with the first adjacency definition. Thus, for clearcuts, we consider that two stands are adjacent if they share a boundary of positive length (*strong* adjacency). For habitats, we consider that it is sufficient to share at least one single point (*weak* adjacency) (Goycoolea et al., 2009). Figure 5 shows the application of the two different adjacency definitions.

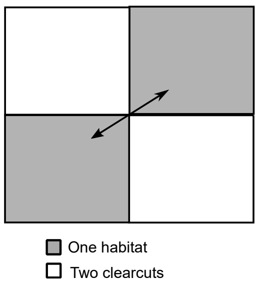

Figure 5: *Strong* adjacency for clearcuts and *weak* adjacency for habitats.

Let $G = (\mathcal{V}, \mathcal{E})$ be a graph where each vertex in $\mathcal{V} = \{1, \ldots, n\}$ corresponds to a stand of the forest, and the vertices of each edge in $\mathcal{E}$ correspond to two adjacent stands according to strong adjacency. As a consequence, the graph is planar, *i.e.*, it can be drawn in a plane surface without crossing edges. Let $\mathcal{T} = \{1, 2, \ldots, T\}$ represent the planning horizon. Let $\mathcal{K}$ be the set of maximal cliques. A clique is a set of vertices of a complete subgraph of the graph, *i.e.*, there is an edge between each pair of vertices, and it is maximal if it is not contained in any other clique (see Goycoolea et al. (2009) for a procedure to generate $\mathcal{K}$). As a planar graph does not contain cliques with five vertices or more (Diestel, 2012), $\mathcal{K}$ can be found in a polynomial number of calculations.

Notation. We define the following notation.

Indices:

t, u - period identifiers;

i, j - stand identifiers;

q - clique identifier;

r - subregion identifier;

c - subset identifier corresponding to a cluster that may be harvested;

h - subset identifier corresponding to a cluster that may be qualified as a habitat.

Data:

$\mathsf{A}^{\mathsf{max}}$ - maximum clearcut area;

$\mathsf{H}^{\mathsf{min}}$ - minimum area of a habitat;

$\mathsf{C}^{\mathsf{min}}$ - minimum core area of a habitat;

$\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$ - minimum total habitat area in each period;

$\mathsf{C}^{\mathsf{min}}_{\mathsf{tot}}$ - minimum total core area in each period;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$ - minimum harvest age, in periods;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ - minimum mature age, in periods;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$ - minimum average age of a stand, in periods, at the end of the planning horizon;

Age_{i0} - age, in periods, of stand i in the period before the beginning of the planning horizon;

The next sets are defined assuming that no intervention is carried out in the forest:

$\mathcal{C}_t$ - set of all possible clusters of stands that in t are not younger than $\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$, such that the area of each cluster is less than or equal to $\mathsf{A}^{\mathsf{max}}$; the model selects clusters of this type to be harvested in such a way that they become clearcuts (maximal harvested clusters);

$\mathcal{H}_t$ - set of all possible clusters of stands that in t are not younger than $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$, such that the area of each cluster is not less than $\mathsf{H}^{\mathsf{min}}$; the model selects clusters of this type to remain mature in such a way that their core areas are not less than $\mathsf{C}^{\mathsf{min}}$;

$\mathcal{V}_t$ - set of stands in period t that are not younger than $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$;

$\mathcal{R}_i$ - set of subregions inside stand i (host stand);

$\mathcal{I}_r$ - set of stands that determine whether subregion r is core area;

$\mathsf{s}_i, \mathsf{s}_c, \mathsf{s}_h, \mathsf{s}_{ir}$ - areas of stand i, clusters c and h, and subregion $r \in \mathcal{R}_i$, respectively;

npv_{ct} - net present value of timber provided by cluster $c \in \mathcal{C}_t$ if it is harvested in period t;

v_{ct} - volume of timber provided by cluster $c \in \mathcal{C}_t$ if it is harvested in period t;

v_0 - target volume of timber to be harvested in each period;

α - deviation allowed from target volume of timber to be harvested;

F - total area of the forest.

Variables: three types of variables are defined:

$$z_{ct} = \begin{cases} 1 & \text{if cluster } c \in \mathcal{C}_t \text{ is harvested in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ht} = \begin{cases} 1 & \text{if cluster } h \in \mathcal{H}_t \text{ is habitat in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$w_{irt} = \begin{cases} 1 & \text{if subregion } r \in \mathcal{R}_i \text{ is core habitat in period } t \\ 0 & \text{otherwise.} \end{cases}$$

Formulation. The formulation of P as a linear integer program is the following:

$$\text{maximize} \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}_t} \mathsf{npv}_{ct} z_{ct} \tag{1}$$

subject to:

$$\sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}_t \colon i \in c} z_{ct} \leq 1, \ \forall i \in \mathcal{V} \tag{2}$$

$$\sum_{c \in \mathcal{C}_t} \mathsf{v}_{ct} z_{ct} \geq (1 - \alpha)\mathsf{v}_0, \ \forall t \in \mathcal{T} \tag{3}$$

$$\sum_{c \in \mathcal{C}_t} \mathsf{v}_{ct} z_{ct} \leq (1 + \alpha)\mathsf{v}_0, \ \forall t \in \mathcal{T} \tag{4}$$

$$\mathsf{F}\ (T+1) - \sum_{t \in \mathcal{T}} t \sum_{c \in \mathcal{C}_t} \mathsf{s}_c z_{ct} + \sum_{i \in \mathcal{V}} \mathsf{s}_i (1 - \sum_{t \in \mathcal{T}} \sum_{\substack{c \in \mathcal{C}_t \colon \\ i \in c}} z_{ct}) \mathsf{Age}_{i0} \geq \mathsf{F}\ \mathsf{Age}_{\mathsf{end}}^{\mathsf{min}} \tag{5}$$

$$\sum_{c \in \mathcal{C}_t \colon c \cap q \neq \emptyset} z_{ct} \leq 1, \ \forall q \in \mathcal{K}, \ \ t \in T \tag{6}$$

$$\sum_{u=tmin}^{t} \sum_{c \in \mathcal{C}_u \colon i \in c} z_{cu} + \sum_{h \in \mathcal{H}_t \colon i \in h} y_{ht} \leq 1, \ \forall t \in \mathcal{T}, i \in \mathcal{V}_t \tag{7}$$

$$w_{irt} + \sum_{\substack{c \in \mathcal{C}_t \colon \\ j \in c}} z_{ct} \leq 1, \ \forall t \in \mathcal{T}, i \in \mathcal{V}_t, r \in \mathcal{R}_i, j \in \mathcal{I}_r \setminus \{i\} \tag{8}$$

$$w_{irt} \leq \sum_{h \in \mathcal{H}_t \colon i \in h} y_{ht}, \ \forall t \in \mathcal{T}, i \in \mathcal{V}_t, r \in \mathcal{R}_i \tag{9}$$

$$\sum_{i \in h} \sum_{r \in \mathcal{R}_i} \mathsf{s}_{ir} w_{irt} \geq \mathsf{C}^{\mathsf{min}} y_{ht}, \ \forall t \in \mathcal{T}, h \in \mathcal{H}_t \tag{10}$$

$$\sum_{i \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_i} \mathsf{s}_{ir} w_{irt} \geq \mathsf{C}^{\mathsf{min}}_{\mathsf{tot}}, \ \forall t \in \mathcal{T} \tag{11}$$

$$\sum_{h \in \mathcal{H}_t} \mathsf{s}_h y_{ht} \geq \mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}, \ \forall t \in \mathcal{T} \tag{12}$$

$$z_{ct} \in \{0,1\}, \ \forall t \in \mathcal{T}, c \in \mathcal{C}_t \tag{13}$$

$$y_{ht} \in \{0,1\}, \ \forall t \in \mathcal{T}, h \in \mathcal{H}_t \tag{14}$$

$$w_{irt} \in \{0,1\}, \ \forall t \in \mathcal{T}, i \in \mathcal{V}_t, r \in \mathcal{R}_i. \tag{15}$$

Expression (1) maximizes the net present value of timber harvested. Constraints (2) ensure that each stand is harvested at most once in the planning horizon. Constraints (3) and (4) require minimum and maximum volumes of timber harvested in each period, respectively. Constraint (5) requires the weighted average age of a stand at the end of the planning horizon, in period $T+1$, to be at least $\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$ periods. Observe that, as clearcuts do not overlap during the planning horizon (each stand is harvested once at the most), the sum of the ages of the stands which have been harvested, weighted by their areas, can be computed as $\sum_{t \in \mathcal{T}} [(T+1) - t] \sum_{c \in \mathcal{C}_t} \mathsf{s}_c z_{ct}$. Furthermore, the sum of the weighted ages of the remaining stands can be computed as $\sum_{i \in \mathcal{V}} \mathsf{s}_i [(T+1) + \mathsf{Age}_{i0}](1 - \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}_t : i \in c} z_{ct})$. Summing these two expressions gives the left-hand-side of constraint (5). Constraints (6) guarantee that if a potential cluster is harvested in a certain period then no adjacent potential clusters (clusters intersecting the same clique) are harvested in this period. This prevents the formation of clearcuts whose areas are greater than $\mathsf{A}^{\mathsf{max}}$, because the area of each cluster is already less than or equal to $\mathsf{A}^{\mathsf{max}}$. Constraints (7) ensure that each stand belongs at the most to one cluster selected either to be harvested or to be habitat. If a stand is selected to belong to a habitat in period t, it can not be selected to be harvested from period $u = \textit{tmin}$ to t, where $\textit{tmin} = \max\{1, t - (\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}} - 1)\}$, because a harvested stand will be mature after $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ periods (and harvesting occurs at the beginning of the periods).

Constraints (8) are relative to the non-inner subregions r inside each mature stand i (when $\mathcal{I}_r \setminus \{i\} \neq \emptyset$), while constraints (9) are relative to all mature subregions. For each period, constraints (8) ensure that subregion r is not core area when an invasive stand $j \in \mathcal{I}_r \setminus \{i\}$ is harvested. Constraints (9) guarantee the same status for subregion r of i when there is no habitat with i. Constraints (9) also prevent an inner subregion from being core area when the host stand is not a piece of a habitat. Observe that sum $\sum_{c \in \mathcal{C}_t : j \in c} z_{ct}$ is over all clusters in $\mathcal{C}_t$ which include j. If this sum is equal to 1 (*i.e.*, a cluster with j is harvested) then, by constraints (8), $w_{irt} = 0$ (implying that r is not core area in t). Observe also that the summation $\sum_{h \in \mathcal{H}_t :\, i \in h} y_{ht}$ is over all clusters in $\mathcal{H}_t$ with i; if this sum is zero (meaning that no clusters with i are habitats) then, by constraints (9), $w_{irt} = 0$ for all subregions r inside i (implying that r is not core area in t).

Constraints (10) ensure the minimum core area requirement for each habitat. The left-hand-side of constraint (10) for a cluster $h \in \mathcal{H}_t$ represents the core area of h. If h is selected to be a habitat ($y_{ht} = 1$), then its core area cannot be less than $\mathsf{C}^{\mathsf{min}}$. Constraints (11) guarantee the minimum requirement of core habitat in each period (for period t, the left-hand-side of constraint (11) represents the total core area in t). For each period, constraints (12) impose a minimum on total habitat area (for period t, the left-hand-side of constraint (12) represents the total habitat area available in t).

Example 1 (continued). *For the sake of simplicity, consider constraints* (8)–(12) *with respect to the first period. Let* $\mathsf{A}^{\mathsf{max}} = 2\,\mathrm{ha}$*,* $\mathsf{H}^{\mathsf{min}} = 1.5\,\mathrm{ha}$*,* $\mathsf{C}^{\mathsf{min}} = 0.75\,\mathrm{ha}$*,* $\mathsf{C}^{\mathsf{min}}_{\mathsf{tot}} = 1\,\mathrm{ha}$ *and* $\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}} = 2\,\mathrm{ha}$*. Thus, taking into account that the area of each stand is* $1\,\mathrm{ha}$ *(Table 1), the set of possible clusters is* $\mathcal{C}_1 = \{A, B, C, AB, AC, BC\}$ *and the set of potential habitats is* $\mathcal{H}_1 = \{AB, AC, BC, ABC\}$*.*

Table 2 lists indices i, r, j for constraints (8)*.*

i	r	j	i	r	j	i	r	j
A	A_2	C	B	B_2	C	C	C_2	A
A	A_3	B	B	B_3	A	C	C_3	B
A	A_4	B	B	B_4	A	C	C_4	A
A	A_4	C	B	B_4	C	C	C_4	B

Table 2: Indices i, r, j for constraints (8).

- *Take the case of stand B. If $z_{B,1} = 1$ (B is harvested), then, by constraints* (8)*, $w_{i,r,1} = 0$ for all the following (i, r) pairs (Table 2): (A, A_3), (A, A_4), (C, C_3), (C, C_4). That is, subregions r in these pairs are not core area. Furthermore, by constraints* (7)*, $\sum_{h \in \mathcal{H}_1:\, B \in h} y_{h,1} = 0$ (AB, BC, ABC are not habitats). By constraints* (9) *on stand B, $w_{B,r,1} = 0$ for all subregions r inside B. However, clusters in $\mathcal{H}_1$ with A or C have to be selected ($\sum_{h \in \mathcal{H}_1:\, A \in h} y_{h,1} = 1$ or $\sum_{h \in \mathcal{H}_1:\, C \in h} y_{h,1} = 1$), otherwise, constraints* (12) *would be violated. Constraints* (11) *would also be violated, because, by constraints* (9)*, $w_{i,r,1} = 0$ for (i, r) pairs (A, A_1), (A, A_2), (A, A_3), (A, A_4), (C, C_1), (C, C_2), (C, C_3) and (C, C_4). Thus, $y_{A,C,1} = 1$. To satisfy constraints* (10)*, either $w_{A,A_1,1} = 1$ or $w_{C,C_1,1} = 1$. To satisfy constraints* (11)*, there must be $w_{A,A_1,1} = w_{C,C_1,1} = 1$ (variables $w_{A,A_2,1}$ and $w_{C,C_2,1}$ may assume values zero or one). Constraints* (12) *are satisfied, because the total habitat area* ($2\,\mathrm{ha}$) *is greater than or equal to* $\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$*.*

  By constraints (7)*, $\sum_{c \in \mathcal{C}_1: A \in c} z_{c,1} = \sum_{c \in \mathcal{C}_1: C \in c} z_{c,1} = 0$, that is, A and C are not harvested.*

- *Take the case of stands B and C. If $z_{B,1} = z_{C,1} = 1$ (B and C are harvested), then, by constraints* (8)*, $w_{ir,1} = 0$ for all (i, r) pairs in Ta-*

ble 2 except (B, B_3) and (C, C_2). By constraints (7)*,* $\sum_{h\in\mathcal{H}_1:\, B\in h} y_{h,1} = \sum_{h\in\mathcal{H}_1:\, C\in h} y_{h,1} = 0$*. By constraints* (9) *on stands B and C,* $w_{B,r,1} = 0$ *and* $w_{C,r,1} = 0$ *for all subregions r inside B and C, respectively. Therefore, A_1 is the only subregion that can be core area. If* $w_{A,A_1,1} = y_{A,1} = 1$*, constraints* (10) *are satisfied (the core area of habitat A is* $0.75\,\mathrm{ha}$*), but constraints* (11) *and* (12) *are violated (the total core area is* $0.75\,\mathrm{ha}$ *and the total habitat area is* $1\,\mathrm{ha}$*). Hence, $z_{B,1}$ and $z_{C,1}$ cannot take the value one simultaneously.*

Constraints (13), (14) and (15) state the binary nature of the decision variables. The main drawback of model (1)-(15) is its large number of variables. It may grow exponentially with the number of stands. This precludes the direct use of general purpose solvers for large instances, primarily because they require enormous amounts of storage memory. A relaxation of this model is used by the procedure described next.

4 Branch-and-bound

As the direct use of general purpose solvers to solve the model described above is not possible for large instances, we propose a branch-and-bound procedure for tackling it.

Branch-and-bound can be used as an exact method or as a heuristic, if it is interrupted on the base of, *e.g.*, a time limit. In the latter case, a bound on the value of the optimal solution can be provided, giving a measure of quality of the solution obtained.

In Neto et al. (2013) a similar branch-and-bound algorithm was proposed for a related problem with non-linear constraints; for almost all instances considered, the method was able to find very good solutions (within 1% of the optimum), or the optimum.

4.1 General description

The main steps of a branch-and-bound procedure are presented in an abridged way in Algorithm 1. The variables used in this algorithm concern the binary decision of harvesting or not a given stand in a given period, which leads to two branches for each node in the search tree. At a node k in the branch-and-bound procedure, solution x is an integer vector where x_i^k is the period in which stand i is harvested (or null if it is not harvested); instantiating it requires fixing some of the variables of P with $x_i^k = \sum_{t\in\mathcal{T}} \sum_{c\in\mathcal{C}_t: i\in c} t z_{ct}$. Hence, a subproblem in the branch-and-bound process corresponds to a relaxation of P (eliminating or weakening some of its constraints) with extra constrains imposing the branching decisions, and where the habitat constraints are not considered. The upper bound on the optimal value of each subproblem is used to prune nodes of the branch-and-bound tree. Indeed, when the upper bound is not greater than the best known net present value, no improvement on the solution value is possible. Each of the main steps of this algorithm will be described in detail in the next sections.

4.2 Initialization

Branch-and-bound makes use of two queues Q and Q' of nodes to explore. At the beginning, Q' is an empty queue. Q is initialized with the root node, defined by the following elements:

- set $\mathcal{S}_0$ of all pairs (stand i, period t) such that i is available to be harvested in t (it is not younger than $\mathsf{Age}_{\mathsf{cut}}^{\mathsf{min}}$ in t, and its area is not greater than $\mathsf{A}^{\mathsf{max}}$), sorted by descending order of the net present value corresponding to i and t;
- solution x^0 where no decision is taken ($x_i^0 = 0$ for all stands i);
- root subproblem P^0.

Root problem P^0 is a relaxation of problem P. Since the set of potential habitats $\mathcal{H}_t$ can be extremely large, the habitat constraints (7), (9), (10) and (14) are removed. Constraints (12), which impose a minimum in the total habitat area in each period, are replaced by constraints (16), that simply ensure, in each period, that the area of mature stands is not less than the minimum required habitat area $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}$:

$$M_t - \sum_{u=tmin}^{t} \sum_{c \in \mathcal{C}_u} \mathsf{s}_c z_{cu} \geq \mathsf{H}_{\mathsf{tot}}^{\mathsf{min}},\ \forall t \in \mathcal{T}, \tag{16}$$

where M_t is the area of all mature stands in period t, s_c is the area of stands in $c \in \mathcal{C}_u$ that are not younger than $\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}}$ in period t, and *tmin* $= \max\{1, t - (\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}} - 1)\}$. This defines the root subproblem P^0, whose initial solution x^0 is a null vector (no branching decision is taken). These values are used to initialize queue Q, as the root node.

Q' corresponds to nodes k for which the solver has not been able to find an upper bound for subproblem P^k (relaxation of P) within the time limit allowed for the solution of subproblems. These nodes may be explored later, but only after the main queue Q is empty. This scheme permits limiting the time that is allocated to an initial exploration of each node, without excluding optimality, if time allows.

4.3 Termination

Branch-and-bound terminates normally, with the optimal solution (or with the knowledge that the instance is infeasible), when both queues Q and Q' are empty. Otherwise, it terminates as it exceeds the time allowed to solve the instance, *CPULIM*; in this case, the method will propose the best solution found. It may happen that no feasible solution has been found within time *CPULIM*. In such a situation the method simply fails, with no solution to propose, even though the instance has not been shown to be infeasible.

Algorithm 1: Main steps of the branch-and-bound algorithm

Step 1 **Initialization:**
create empty queues Q and Q'
set best net present value $\mathsf{npv}^* = -\infty$.
solve subproblem P^0 (root relaxation)
if *solution of subproblem* P^0 *is optimal* **then**
  if *solution of* P^0 *is feasible for the original problem* **then**
    update $\mathsf{npv}^* :=$ optimum of P^0
  else add root relaxation node to Q

Step 2 **Termination:**
if $Q = \emptyset$ **and** $Q' = \emptyset$ **then**
  if $\mathsf{npv}^* = -\infty$ **then** the problem is infeasible
  else solution x that yielded npv^* is optimal
if *time limit has been reached* **then**
  if $\mathsf{npv}^* = -\infty$ **then** nothing can be said
  else propose heuristic solution x that yielded npv^*

Step 3 **Problem selection and relaxation:**
if $Q \neq \emptyset$ **then** select and remove a node k from Q
else select and remove a node k from Q'
if *solution x proven to be infeasible* **then** go to Step 2
solve subproblem P^k (current relaxation) with a limit on time
if *a feasible solution has been found* **then**
  let LB^R, UB^R be lower and upper bounds obtained for this subproblem
else
  if *subproblem not proven to be infeasible* **then** add current node to Q';
  go to Step 2

Step 4 **Pruning:**
if $UB^R \leq \mathsf{npv}^*$ **then** go to Step 2
if $\mathsf{npv}(x) > \mathsf{npv}^*$ *and there is no stand, period pair to fix* **then**
  update $\mathsf{npv}^* := \mathsf{npv}(x)$
  delete dominated nodes from Q and Q'
  go to Step 2;

Step 5 **Partitioning:**
choose a stand, period pair not yet fixed, and create two branches: one where the stand is cut in that period, another where it isn't
update solution x and net present value $\mathsf{npv}(x)$ in the two nodes
insert the two nodes in Q
go to Step 2

4.4 Problem selection and relaxation

Branch-and-bound selects and removes a node k from one of the queues; nodes are selected from Q' only if Q is empty. Both Q and Q' are *last-in, first-out* queues; with the order chosen for inserting nodes (see 4.6), this corresponds to a variant of depth-first search.

The infeasibility of partial solution x^k is checked. If it is infeasible, it is immediately rejected; if we could not conclude that it is infeasible, then subproblem P^k is solved, with a limit on time.

Notice that if the solution of P^0 is optimal and it is feasible for the original problem (*i.e.*, all the constraints relaxed from P are satisfied), the solution is optimal for P.

4.5 Pruning

When the upper bound for the current subproblem UB^R is not greater than the currently best known net present value, npv^*, the node is pruned (or fathomed). Let $\mathsf{npv}(x)$ be the net present value of solution x. If $\mathsf{npv}(x)$ is greater than npv^* and all decisions are taken (*i.e.*, all (stand, period) pairs are fixed), then npv^* is updated; furthermore, all nodes in Q or Q' with $UB^R \leq \mathsf{npv}^*$ are pruned.

4.6 Partitioning

Each node corresponds to the decision of harvesting or not a stand in a certain period. Subproblem P^k of node k is obtained from P^0 incorporating all the decisions already taken up from the root to this node. Each node k may lead to two new nodes, corresponding to the decisions of harvesting stand i_k in period t_k (the *left-hand* node, $k+1$) or not (the *right-hand* node, $k+2$). The right-hand node is inserted first into queue Q or Q'. The tuple (i_k, t_k) chosen for partitioning corresponds to the first element of set $\mathcal{S}_k$.

The new partial solutions x^{k+1} and x^{k+2}, corresponding to the decisions of harvesting or not stand i_k in period t_k (left and right branches), are obtained from x^k as follows:

- for x^{k+1}, we fix $x_{i_k}^{k+1} = t_k$, and $x_i^{k+1} = x_i^k$ for all $i \neq i_k$;
- for x^{k+2}, we let $x_i^{k+2} = x_i^k$ for all i.

The net present value of x^{k+1} is equal to that of x^k plus the net present value of stand i_k in period t_k; x^{k+2} has the same net present value as x^k. Sets $\mathcal{S}_{k+1}$ and $\mathcal{S}_{k+2}$, corresponding to the two new branches, are initialized by removing (i_k, t_k) from $\mathcal{S}_k$; $\mathcal{S}_{k+1}$ is updated by removing the remaining pairs that contain i_k. It is also removed any pair (i, t) such that harvesting stand i in period t violates any the following constraints: upper bound on the volume of timber harvested in each period; ending-age constraint; for all i adjacent to i_k , maximum clearcut area.

This contributes to a significant reduction of the branch-and-bound tree, which is attractive especially because the cost of checking these constraints is low.

Fixing decisions in the subproblems is performed as follows. Let $\mathcal{L}_t^k$ be the set of clearcuts in period t and $\mathcal{J}_k$ the set of pairs (i,t) such that stand i is harvested in t ($i \in c$ for some $c \in \mathcal{L}_t^k$). Let $\mathcal{N}_k$ be the set of pairs (i,t) such that i is not harvested in t ($\mathcal{N}_k = \mathcal{S}_0 \backslash (\mathcal{S}_k \cup \mathcal{J}_k)$). The following constraints are introduced, corresponding to the decisions already taken up to node k:

$$\sum_{c \in \mathcal{C}_t:\ c \cap c' = c'} z_{ct} = 1,\ \forall c' \in \mathcal{L}_t^k, \forall t \in \mathcal{T} \tag{17}$$

$$\sum_{c \in \mathcal{C}_t:\ i \in c} z_{ct} = 0,\ \forall (i,t) \in \mathcal{N}_k. \tag{18}$$

Constraints (17) impose that for each clearcut c' in each period, just one potential clearcut containing c' (or simply c') must be harvested in the period. Constraints (18) impose that for each stand that is not harvested in a certain period, no potential clearcut containing the stand is harvested in this period.

Subproblems P^{k+1} and P^{k+2} are obtained from P^k with the following updates:

- for the left branch P^{k+1}:
  - if stand i_k is added to some clearcut $c'' \in \mathcal{L}_{t_k}^k$, constraint (17) for c'' must be replaced by the one corresponding to the new clearcut. Otherwise, constraint (17) for $c' = \{i_k\}$ must be added; $\mathcal{L}_{t_k}^{k+1}$ is obtained from $\mathcal{L}_{t_k}^k$ according to these changes;
  - $\mathcal{J}_{k+1} = \mathcal{J}_k \cup \{(i_k, t_k)\}$;
  - $\mathcal{N}_{k+1} = \mathcal{N}_k \cup \mathcal{S}_k \backslash (\mathcal{S}_{k+1} \cup \mathcal{J}_{k+1})$;
- for the right branch P^{k+2}:
  - $\mathcal{L}_t^{k+2} = \mathcal{L}_t^k, \forall t$;
  - add constraint (18) for (i_k, t_k);
  - $\mathcal{J}_{k+2} = \mathcal{J}_k$;
  - $\mathcal{N}_{k+2} = \mathcal{N}_k \cup \mathcal{S}_k \backslash (\mathcal{S}_{k+2} \cup \mathcal{J}_{k+2}) = \mathcal{N}_k \cup \{(i_k, t_k)\}$.

5 Computational experiment

5.1 Instances

We tested instances available for download at the FMOS Dataset[1]: *El Dorado*, El Dorado National Forest in northern California, the USA (referred to in Goycoolea

[1] `http://ifmlab.for.unb.ca/fmos/datasets/`

et al. 2005); *Stafford*, a forest in British Columbia, Canada; *Kittaning*4, *Bear Town*, *Phyllis Leeper*, and *Five Points*, forests in Pennsylvania, the USA; *WLC*, and the computer generated instances *FLG*9 and *FLG*10 (Paradis and Richards, 2001).

The number of stands ranges from 32 (Kittaning4) to 1363 (El Dorado) and the number of edges from 48 to 4087 (considering weak adjacency). The length of the temporal horizon ranges from 3 to 8 periods. Some instances are tested with a different number of periods T; in such cases, the notation used is *instance*$_T$. Tables 3 and 4 summarize the characteristics of the instances.

Instance	No. stands	Weak adjacency No. edges	Strong adjacency No. edges	Strong adjacency No. cliques	No. periods	No. years per period
El Dorado	1363	4087	3617	2041	3	10
Stafford	1008	2113	2066	1163	3	10
$\text{FLG9}_{3/8}$	850	2524	2388	1420	3 / 8	5
$\text{FLG10}_{3/8}$	763	2262	2137	1269	3 / 8	5
$\text{Five Points}_{3/5}$	90	164	149	88	3 / 5	10
$\text{PhyllisLeeper}_{3/5}$	89	161	131	86	3 / 5	10
$\text{WLC}_{3/7}$	73	114	98	63	3 / 7	5
$\text{Bear Town}_{3/5}$	71	148	101	64	3 / 5	10
$\text{Kittaning4}_{3/5}$	32	48	47	25	3 / 5	10

Table 3: Size of the instances.

Values for α, v_0, $\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$ are fixed as in Neto et al. (2013). Parameter α, the maximum deviation of the harvested volume in each period from the target volume, is set to 0.15 and parameter v_0, the target volume for each period, is calculated as the timber volume of all stands in the first period divided by the total number of periods, that is, $\mathsf{v}_0 = \sum_{i=1}^{n} \mathsf{v}_{i1}/T$ (see, *e.g.*, Yoshimoto and Brodie, 1994). In order to obtain a feasible solution, Neto et al. (2013) replaced the denominator of v_0 by $T+1$ for Bear Town$_5$, WLC$_3$ and PhyllisLeeper$_5$, $T+2$ for Bear Town$_3$, PhyllisLeeper$_3$, FLG9$_8$ and FLG10$_8$, Stafford and El Dorado, and $T+3$ for FLG9$_3$ and FLG10$_3$. The minimum average age for the forest at the end of the planning horizon $\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$ is set to $(T+1) - T/2$.

In order to obtain a feasible solution, or to make the constraints of total core area or total habitat area active, different values for the minimum total habitat area in each period $\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$ and for the minimum total core area in each period $C^{\min}_{\text{tot}}$ are used. Table 4 shows the values of v_0, $\mathsf{Age}^{\mathsf{avg}}_1$ (average age of the stands in the first period), $\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$, $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$, $\mathsf{A}^{\mathsf{max}}$, $\mathsf{C}^{\mathsf{min}}$, F, $\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$ and $\mathsf{C}^{\mathsf{min}}_{\mathsf{tot}}$ (in percentage of F).

We tested three impact zone widths: $50\,\mathrm{m}$, $100\,\mathrm{m}$ and $150\,\mathrm{m}$. Tables 5 and 6 describe the subregions of the forests according to these values.

As the impact zone width increases, the number of subregions increases and the average area of subregions decreases for all instances except for FLG9$_{3/8}$ from $50\,\mathrm{m}$ to $100\,\mathrm{m}$. The average number of stands of sets $\mathcal{I}_r$ (that determine whether a subregion is core area) increased slightly for the majority of the instances. The number and average area of inner subregions decreased significantly. Therefore,

Instance	v_0 m^3)	$\mathsf{Age}_1^{\mathsf{avg}}$ (years)	$\mathsf{Age}_{\mathsf{cut}}^{\mathsf{min}}$ (years)	$\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}}$ (years)	$\mathsf{A}^{\mathsf{max}}$ (ha)	$\mathsf{C}^{\mathsf{min}}$ (ha)	F (ha)	$\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}$ %	$\mathsf{C}_{\mathsf{tot}}^{\mathsf{min}}$ %
El Dorado	683632	105.86	60	80	40	40	21147	10	5
Stafford	668558	50.83	60	60	40	40	10444	10	7.5
$\text{FLG9}_{3/8}$	114149 / 76099	31.91	40	40	46	46	10000	5	2.5
$\text{FLG10}_{3/8}$	102638 / 68425	27.02	40	40	46	46	10000	5	2.5
$\text{Five Points}_{3/5}$	426 / 256	63.11	60	60	40	40	677	10	5
$\text{Phyllis Leeper}_{3/5}$	4437 / 3169	94.38	80	80	40	40	646	10	5
$\text{WLC}_{3/7}$	18415 / 10523	46.58	40	40	40	40	897	10	4.5
$\text{Bear Town}_{3/5}$	3468 / 2890	95.49	80	80	40	40	546	10	5
$\text{Kittaning4}_{3/5}$	695 / 417	66.59	60	60	20	20	238	15	10

Table 4: Values of v_0, $\mathsf{Age}_1^{\mathsf{avg}}$ (average age of the stands in the first period), F (area of the forest) and parameters $\mathsf{Age}_{\mathsf{cut}}^{\mathsf{min}}$ (minimum harvest age), $\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}}$ (minimum mature age), $\mathsf{A}^{\mathsf{max}}$ (maximum clearcut area), $\mathsf{C}^{\mathsf{min}}$ (minimum core area for a habitat), $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}$ (minimum total habitat area in each period, as a percentage of F) and $\mathsf{C}_{\mathsf{tot}}^{\mathsf{min}}$ (minimum total core area in each period, as a percentage of F).

	Subregions								
	50 m			100 m			150 m		
Instance	No.	Average area (ha)	Ave. no. stands	No.	Ave. area (ha)	Ave. no. stands	No.	Ave. area (ha)	Ave. no. stands
El Dorado	19910	1.06	2	25617	0.82	3	31359	0.06	3
Stafford	10560	0.99	2	13826	0.76	3	17403	0.60	3
$\text{FLG9}_{3/8}$	10151	0.99	2	9965	1.00	2	15222	0.66	3
$\text{FLG10}_{3/8}$	9129	1.10	2	9154	1.09	2	13442	0.74	3
$\text{Five Points}_{3/5}$	1347	0.50	3	1680	0.40	3	1962	0.35	4
$\text{Phyllis Leeper}_{3/5}$	961	0.67	2	1307	0.49	3	1542	0.42	4
$\text{WLC}_{3/7}$	675	1.33	2	1201	0.75	3	1395	0.64	4
$\text{Bear Town}_{3/5}$	776	0.70	2	1043	0.52	3	1225	0.45	4
$\text{Kittaning4}_{3/5}$	337	0.71	2	404	0.59	3	479	0.50	4

Table 5: Description of the forests in terms of subregions according to the impact zone width (to be continued).

			Inner subregions					
	Stands		50 m		100 m		150 m	
Instance	No.	Average area (ha)	No.	Average area (ha)	No.	Average area (ha)	No.	Average area (ha)
El Dorado	1363	15.52	1355	6.49	897	3.61	438	0.23
Stafford	1008	10.36	992	5.19	827	2.53	464	1.65
FLG9$_{3/8}$	850	11.76	704	7.03	535	3.62	347	1.33
FLG10$_{3/8}$	763	13.10	629	8.21	517	4.18	354	1.64
Five Points$_{3/5}$	90	7.52	88	2.48	49	0.72	7	0.07
Phyllis Leeper$_{3/5}$	89	7.26	87	2.77	57	0.94	16	0.57
WLC$_{3/7}$	73	12.28	61	7.52	49	3.86	25	2.56
Bear Town$_{3/5}$	71	7.69	71	2.83	48	0.99	14	0.76
Kittaning4$_{3/5}$	32	7.44	31	2.85	20	1.17	6	0.78

Table 6: Description of the forests in terms of subregions according to the impact zone width (conclusion).

we may say that the number of non-inner subregions increased for almost all instances. For FLG9$_{3/8}$, fewer and larger non-inner subregions were observed from $50\,\mathrm{m}$ to $100\,\mathrm{m}$ width. For all instances, the average area of the inner subregions was significantly smaller than the average area of stands for $50\,\mathrm{m}$, and decreased as the width of impact zone increased.

5.2 Computational results

The platform used was an Intel Core i7 quad-core CPU, running at $3.4\,\mathrm{GHz}$ in a Mac OS X version 10.10.2, with $24\,\mathrm{GB}$ of RAM. All software was implemented in the Python language, version 2.7.9, using Cplex version 12.6. The branch-and-bound procedure was allowed to run for two hours at the most. The parameter ϵ used to measure solution quality was set to 10^{-4} and, thus, a solution within 0.01% of the optimum is considered optimal.

Table 7 shows, according to the impact zone width, the total time of branch-and-bound, the time required to find the best known incumbent and the quality of this solution (gap^*). The quality of the best solution is measured by using the deviation (in percentage) of its net present value npv^* from the maximum upper bound bub over all unexplored nodes ($\mathrm{gap}^* = (\mathrm{bub} - \mathsf{npv}^*)/\mathsf{npv}^* \times 100$). In some cases the solving time for root subproblem is larger than the maximum allowed CPU time; in these cases, the method stops immediately after solving just the root node. The optimum was found with all impact zone widths for the medium size instances Phyllis Leeper$_3$, Phyllis Leeper$_5$ ($50\,\mathrm{m}$ and $100\,\mathrm{m}$), Bear Town$_3$ ($50\,\mathrm{m}$) and for the smallest instance Kittaning4$_{3/5}$. For the large instance El Dorado the optimum was found with the impact zone widths of $50\,\mathrm{m}$ and $100\,\mathrm{m}$. For another large instance, Stafford, the solution gaps were within 1%. For FLG9$_{3/8}$ and FLG10$_{3/8}$ the solution gaps were slightly above 7%.

One aspect that is worth mentioning is that the quality of the non-optimal solutions found can be better than gap values reveal. In fact, the upper-bounds used might be weak, partly because some constraints of problem P were not considered in their computation.

	50 m			100 m			150 m		
Instance	best solution time (s)	total time (s)	gap* (%)	best solution time (s)	total time (s)	gap* (%)	best solution time (s)	total time (s)	gap* (%)
El Dorado	1176	1176	0.00	1727	1727	0.01	9656	9656	0.07
Stafford	6168	7200	0.56	4912	7200	0.58	3676	7200	0.57
$FLG9_3$	7168	7200	8.63	6918	7200	8.66	6850	7200	8.82
$FLG9_8$	6767	7200	8.21	6908	7209	8.21	105	7200	8.19
$FLG10_3$	6318	7200	7.87	6654	7200	8.03	6314	7200	8.23
$FLG10_8$	7048	7200	7.72	7021	7200	7.70	6155	7200	7.73
$FivePoints_3$	7200	7200	0.26	7200	7200	0.24	7200	7200	0.26
$FivePoints_5$	7200	7200	0.62	7200	7200	0.59	7200	7200	0.61
$PhyllisLeeper_3$	1007	1007	0.01	950	950	0.01	392	392	0.01
$PhyllisLeeper_5$	3141	3141	0.01	7029	7029	0.01	7200	7200	0.03
WLC_3	7091	7200	0.30	7184	7200	0.32	6954	7200	0.39
WLC_7	6752	7200	0.63	6843	7200	0.69	7146	7200	0.68
$BearTown_3$	7200	7200	0.01	7200	7200	0.02	7200	7200	0.02
$BearTown_5$	7200	7200	0.03	7200	7200	0.04	7200	7200	0.02
$Kittaning4_3$	11.60	11.60	$< \epsilon$	11.52	14.68	$< \epsilon$	23.49	27.21	$< \epsilon$
$Kittaning4_5$	0.37	0.37	$< \epsilon$	0.53	0.53	$< \epsilon$	1.00	1.00	$< \epsilon$

Table 7: Computational results of branch-and-bound (time and gap of the best solution found and total time of branch-and-bound) according to the impact zone width; ϵ is set to 10^{-4}.

Table 8 summarizes the effect of impact zone width on the number, total core area and total area of habitats. The values displayed in these tables are relative to the last period of the planning horizon. In general, as expected, for almost all instances, the total core area decreased with larger impact zone widths. For El Dorado there was a significant increase in the total core area with the impact zone width of $150\,\mathrm{m}$ in comparison with impact zone width of $100\,\mathrm{m}$, due to the increase of the number of habitats and, consequently, the total area of habitats.

Table 9 shows the effect of core area constraints on the net present value. We observed that for some of the instances it was not possible to address core area without reducing the attainment of the net present value of timber harvested; this reduction was relatively small for the majority of the instances. However, for some instances the net present value of the best found solution slightly increased (note that solutions are approximate, in these cases). One explanation for the relatively low cost of addressing the habitat availability is that the definition of mature forests (older than 40, 60 or 80 years old) gives a good supply of mature patches over time, providing space for alternative solutions.

Figures 6 and 7 represent the state of forest at the last period, for solutions obtained on instances $PhyllisLeeper_3$ and $Kittaning4_3$, when the parameter defining

Instance	0 m			50 m			100 m			150 m		
	No. habitats	Core area (ha)	Habitat area (ha)	No. habitats	Core area (ha)	Habitat area (ha)	No. habitats	Core area (ha)	Habitat area (ha)	No. habitats	Core area (ha)	Habitat area (ha)
El Dorado	36	6921	6921	33	6168	6949	31	5319	6800	35	6870	7114
Stafford	17	1045	1045	16	989	1045	15	954	1045	15	904	1045
FLG9$_3$	14	1507	1507	12	1436	1501	12	1372	1504	11	1275	1501
FLG9$_8$	13	1502	1502	12	1397	1504	11	1325	1500	10	1202	1501
FLG10$_3$	18	1503	1503	18	1457	1503	18	1425	1506	18	1357	1508
FLG10$_8$	15	1695	1695	13	1488	1606	11	1314	1500	11	1223	1500
FivePoints$_3$	4	559	559	5	544	557	4	416	462	3	413	444
FivePoints$_5$	4	577	577	5	502	510	4	532	554	5	513	536
PhyllisLeeper$_3$	2	146	146	2	150	156	2	146	169	2	130	162
PhyllisLeeper$_5$	2	134	134	2	121	128	1	42	71	2	112	127
WLC$_3$	1	123	123	1	133	138	1	103	119	1	107	134
WLC$_7$	2	138	138	1	152	156	1	92	100	1	100	100
BearTown$_3$	1	113	113	1	113	113	1	107	113	1	96	113
BearTown$_5$	1	103	103	1	110	111	1	96	97	1	69	92
Kittaning4$_3$	2	46	46	2	42	46	2	44	59	2	43	70
Kittaning4$_5$	2	82	82	2	80	82	2	78	82	2	75	82

Table 8: Number, total core area and total area of habitats in the last period, according to the impact zone width ($0\,\mathrm{m}$ means no impact).

Instance	npv^* 0 m	npv^* change (%) 150 m
El Dorado	2509807.23	-0.15
Stafford	137816064.79	$+0.03$
FLG9$_3$	45494316.06	-0.21
FLG9$_8$	53826161.90	$+0.02$
FLG10$_3$	42755260.80	$< \epsilon$
FLG10$_8$	50321195.19	$< \epsilon$
FivePoints$_3$	537437.18	$< \epsilon$
FivePoints$_5$	397854.67	$< \epsilon$
PhyllisLeeper$_3$	5601387.67	$< \epsilon$
PhyllisLeeper$_5$	4963643.20	$< \epsilon$
WLC$_3$	4652363.38	-0.14
WLC$_7$	4207738.19	$< \epsilon$
BearTown$_3$	4381789.22	$< \epsilon$
BearTown$_5$	4528271.47	$< \epsilon$
Kittaning4$_3$	1680889.01	-0.08
Kittaning4$_5$	1452787.82	$< \epsilon$

Table 9: Net present values considering an impact zone width of $0\,\mathrm{m}$ (no impact), and reductions ($-$) or gains ($+$) (in percentage) with the inclusion of core area constraints using an impact zone width of $150\,\mathrm{m}$.

the impact zone width increases from $0\,\mathrm{m}$ (no impact) up to $150\,\mathrm{m}$. In general, there is an influence of this parameter on the arrangement of the habitats. For PhyllisLeeper$_3$, we observe an increase on the total habitat area when the parameter increases from $0\,\mathrm{m}$ up to $100\,\mathrm{m}$. For this instance, from $50\,\mathrm{m}$ up to $150\,\mathrm{m}$ there is a reduction on the total core area. For Kittaning4$_3$ we can observe a similar increase on habitat area from $50\,\mathrm{m}$ up to $150\,\mathrm{m}$.

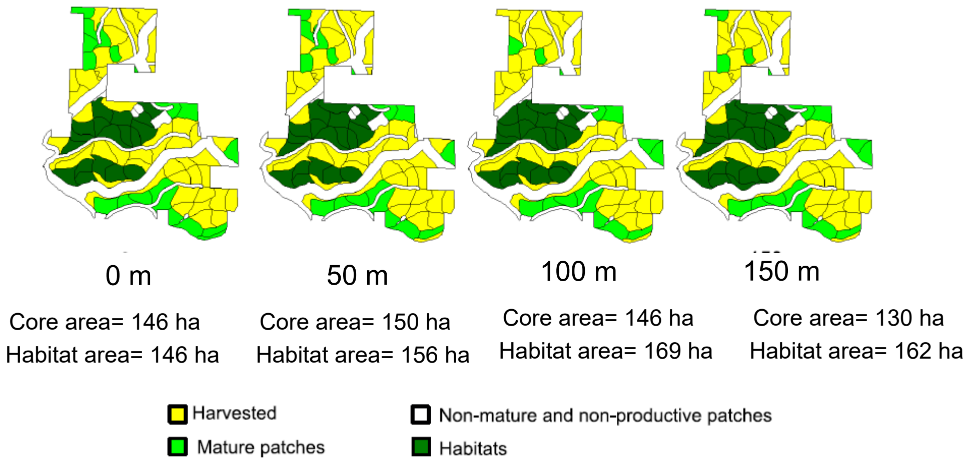

Figure 6: Map representing the solution obtained for PhyllisLeeper$_3$ with impact zone width of $0\,\mathrm{m}$, $50\,\mathrm{m}$, $100\,\mathrm{m}$ and $150\,\mathrm{m}$.

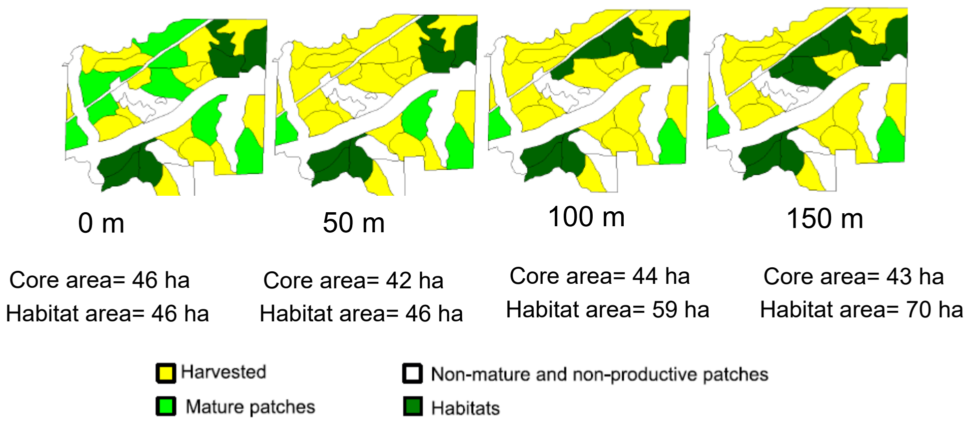

Figure 7: Map representing the solution obtained for Kittaning4$_3$ with impact zone width of $0\,\mathrm{m}$, $50\,\mathrm{m}$, $100\,\mathrm{m}$ and $150\,\mathrm{m}$.

To test the performance of our branch-and-bound procedure, we tried to solve formulation (1)-(15) by Cplex. Cplex solved Kittaning4$_{3/5}$ and WLC$_3$, but it was not able to find a feasible solution within two hours for any of the other instances. For Kittaning4$_{3/5}$, Cplex spent much more time than branch-and-bound procedure (*e.g.*, Cplex required $316\,\mathrm{s}$ to solve Kittaning4_5 with the impact zone width of $150\,\mathrm{m}$, while branch-and-bound required $1\,\mathrm{s}$).

6 Conclusions

Forest fragmentation occurs when large, continuous forests are divided into smaller blocks by land clearing for timber, agriculture, roads, urbanization, or other development. This process reduces the forest's function as a habitat for many plant and animal species. When a forest is fragmented, the amount of edge increases at the expense of core area. Species dependent on core area suffer, while edge-dependent species, including invasive species and predators, thrive.

This study presents a branch-and-bound method designed specifically for forest harvest scheduling problems addressing constraints on clearcut size and core area. Core area is determined by patch area, shape and nature of immediately surrounding conditions. Many indicators have been proposed for assessing core area (*e.g.* ratio perimeter to area), but there is no single indicator that summarizes all characteristics of core area, except those that involving core area itself. In this work, we propose to handle core area directly, using the concept of subregions. Recent research on forest planning problems applying this concept to address core area has not included constraints on maximum clearcut size. We propose an integer programming model for the forestry problem with an exponential number of variables and constraints, based on the so-called cluster formulation, whose relaxations provide bounds to prune nodes throughout the branch-and-bound tree.

Branch-and-bound was tested with forests ranging from 32 to 1363 stands, and temporal horizons ranging from three to eight periods were employed. The main objective of the computational tests was to assess the ability of branch-and-bound to obtain solutions of a certain quality in a reasonable time (up to two hours). We analyzed the impact of some depths of the edge effect (impact zone widths) on the characteristics of the subregions. We also studied the impact of the edge effect on the spatial arrangement of the habitats.

When the size of the impact zone considered increases, the average area of inner subregions (subregions that are core area if the stands where they are included are mature whatever happens to adjacent stands) is significantly smaller than the average area of stands. The number of these subregions greatly decreases with increasing impact zone width.

In general, branch-and-bound was able to find good solutions in a reasonable time. For five instances the optimum was found with all or some impact zone widths, and for other five instances the solution gaps were not greater than 1%. The remaining instances (some of the large instances) showed solution gaps slightly above 7%. For almost all the instances, addressing core area implied small reductions of the net present value. Having a good supply of mature patches over time seems to be possible while keeping mature forests, which might be one explanation for this tendency. The spatial arrangement of habitats was influenced by the impact zone width; in general, a reduction the total core area is associated to larger impact zone widths.

Focusing efforts on modelling and solving methodologies to forest harvest scheduling problems addressing forest fragmentation will help to enhance the ecological value and function of forests managed for timber production. The computational results obtained in this work highlight the merit of the branch-and-bound approach, but further research to improve the ability of the algorithm to solve much larger problems is needed. Further work may also include the study of forestry problems that take into account both core area and inter-habitat connectivity, the most important forest characteristics affected by fragmentation.

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7 APPENDIX - Determining core area

To determine core area, the forest is classified into subregions with the geographic information system ArcGis 9.2. A surrounding impact zone of a given width for each stand is created, using the tool *Buffer*, available in *ArcToolbox* $\backslash$ *Analysis Tools* $\backslash$ *Proximity*. Then, subregions are provided by *ArcToolbox* $\backslash$ *Analysis Tools* $\backslash$ *Overlay*. For each subregion, the defining set and the area are displayed by default. Centroids, useful to distinguish subregions with the same defining set and the same area (see subregions C_4 and C_5 in Figure 8 and Example 2), are also computed, using the tools *Feature to Point* and *Add XY Coordinates* from *ArcToolbox* $\backslash$ *Data Management Tools* $\backslash$ *Features*.

Example 2. *Figure 8 provides another example of a mature forest with three stands,* A*,* B *and* C*, before and after intersecting the stands and impact zones. The sets of stands determining if subregions* C_4, C_5 *are core area are:* $\mathcal{I}_{C_4} = \mathcal{I}_{C_5} = \{C, A, B\}$.

The choice of harvesting a stand (a left branch in the branch-and-bound tree) requires updating the total area and the total core area of habitats. For node k, let H_t^k be a list that contains the existing habitats in period t. Let τ_h be the total area and γ_h be the total core area of habitats $h \in H_t^k$. At node $k+1$, where stand i_k is selected to be harvested in period t_k, updates to $H_{t_k}^k$, $\tau_{t_k}^k$ and $\gamma_{t_k}^k$ are done according to the following three possibilities:

(a) Stand i_k belongs to a habitat $h \in H_{t_k}^k$ and harvesting i_k leads to one smaller patch h' (Figure 9 (a)). Let s_{i_k} be the area of i_k and R be the amount of core area removed, *i.e.*, the core area of h that was inside i_k plus the new edge in h' caused by harvesting i_k; then:

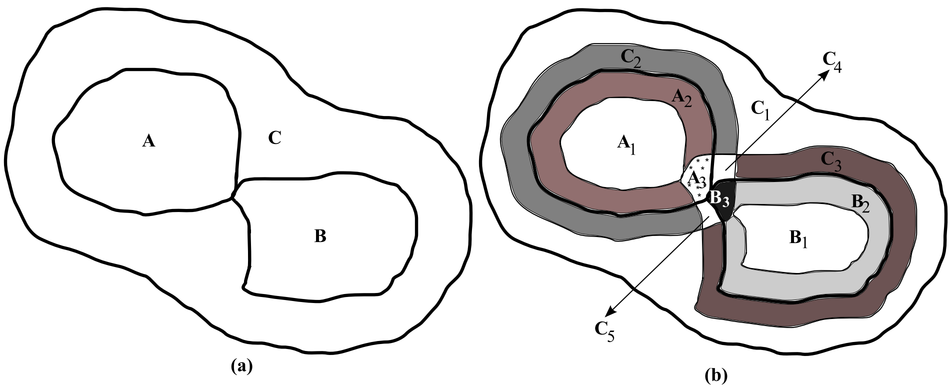

Figure 8: (a) Three-stand mature forest. (b) Subregions.

- If h' meets the minimum core area $\mathsf{C}^{\mathsf{min}}$ requirement for a habitat, then:

$$H_{t_k}^{k+1} = H_{t_k}^{k} \setminus \{h\} \cup \{h'\}$$
$$\gamma_{h'} = \gamma_h - R$$
$$\tau_{h'} = \tau_h - \mathsf{s}_{i_k}$$

- Otherwise, h is removed: $H_{t_k}^{k+1} = H_{t_k}^{k} \setminus \{h\}$.

(b) Stand i_k belongs to habitat $h \in H_{t_k}^{k}$ and harvesting i_k splits up h into a set of new patches $N = \{h_1, \ldots, h_m\}$ (Figure 9 (b)). In this case, it is necessary to calculate the area and the core area of each new patch $h' \in N$, as in (a).

- New habitats are the newly formed patches that meet the requirement for a habitat:

$$H_{t_k}^{k+1} = H_{t_k}^{k} \setminus \{h\} \cup \{h' \in N : \gamma_{h'} \geq \mathsf{C}^{\mathsf{min}}\}$$

(c) Stand i_k (belonging or not to a habitat $h \in H_{t_k}^{k}$) causes edge effects on other habitats $h_1, \ldots, h_m$ (Figure 9 (c)). In this case, the core area of each affected habitat h' is updated by subtracting the new edge caused by harvesting i_k.

- Each h' that becomes non-habitat is removed from $H_{t_k}^{k+1}$.

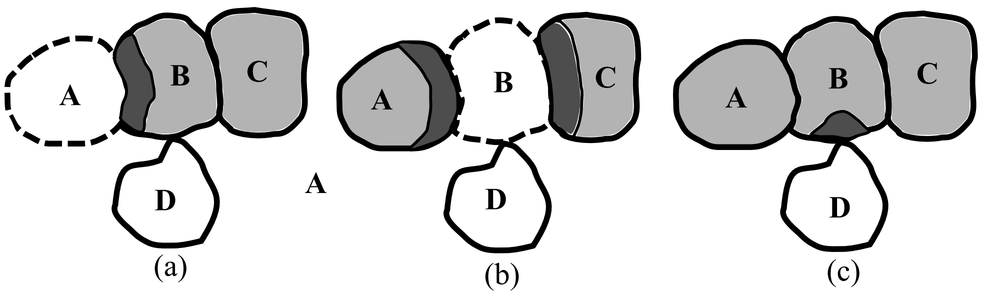

Figure 9: Mature forest with four stands, where patch $\{A, B, C\}$ is habitat. After harvesting stands A (left), B (center) or D (right), the dotted and dark shaded areas are subtracted from the habitat core area, and the dotted area is subtracted from the habitat area.

Chapter 5

A multi-objective Monte Carlo tree search for forest harvest scheduling

This chapter contains the third paper which was submitted to the European Journal of Operational Research, on 6 February 2018.

A multi-objective Monte Carlo tree search for forest harvest scheduling

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Abstract

While the objectives of forest management vary widely and include the protection of resources in protected forests and nature reserves, the primary objective has often been the production of wood products. However, even in this case, forests play a key role in the conservation of living resources. Constraining the areas of clearcuts contributes to this conservation, but if it is too restrictive, a dispersion of small clearcuts across the forest might occur, and forest fragmentation might be a serious ecological problem. Forest fragmentation leads to habitat loss, not only because the forest area is reduced, but also because the core area of the habitats and the connectivity between them decreases. This study presents a Monte Carlo tree search method to solve a bi-objective harvest scheduling problem with constraints on the clearcut area, total habitat area and total core area inside habitats. The two objectives are the maximization of both the net present value and the inter-habitat connectivity. The method is presented as an approach to assist the decision maker in estimating efficient alternative solutions and the corresponding trade-offs. This approach was tested with instances for forests ranging from some dozens to over a thousand stands and temporal horizons from three to eight periods. In general, multi-objective Monte Carlo tree search was able to find several efficient alternative solutions in a reasonable time, even for medium and large instances.

Keywords: Forest planning, Core area, Connectivity, Multi-objective optimization, Tree search.

1 Introduction

Land clearing for timber leads to the so-called *forest fragmentation*, the process of breaking up continuous habitats. This process has many negative impacts on the local environments, mainly on wildlife habitat availability, that is, on habitat area and inter-habitat connections [Harris, 1984, Kurttila et al., 2002]. [Haddad et al., 2015] synthesize results from the set of long-term experiments conducted in a wide variety of ecosystems to demonstrate consistent impacts of fragmentation, how those impacts change over time, and how they align with predictions from theory and observation. This synthesis [1] "revealed strong and consistent responses of organisms and ecosystem processes to fragmentation arising from decreased fragment area, increased isolation, and the creation of habitat edges. Reduced area decreased animal residency within fragments, and increased isolation reduced movement among fragments, thus reducing fragment recolonization after local extinction. Reduced fragment area and increased fragment isolation generally reduced abundance of birds, mammals, insects, and plants. This overall pattern emerged despite complex patterns of increases or declines in abundance of individual species with various proximate causes such as release from competition or predation, shifts in disturbance regimes, or alteration of abiotic factors. Reduced area, increased isolation, and increased proportion of edge habitat reduced seed predation and herbivory, whereas increased proportion of edge caused higher fledgling predation that had the effect of reducing bird fecundity. Perhaps because of reduced movement and abundance, the ability of species to persist was lower in smaller and more isolated fragments. As predicted by theory, fragmentation strongly reduced species richness of plants and animals across experiments, often changing the composition of entire communities. Consistently, all aspects of fragmentation—reduced fragment area, increased isolation, and increased edge—had degrading effects on a disparate set of core ecosystem functions. Degraded functions included reduced carbon and nitrogen retention, productivity, and pollination. In summary, across experiments spanning numerous studies and ecosystems, fragmentation consistently degraded ecosystems, reducing species persistence, species richness, nutrient retention, trophic dynamics, and, in more isolated fragments, movement."

Although constraints on clearcut area were originally imposed in forest management for timber production, mainly to reduce erosion and the negative impacts on aesthetics besides benefitting wildlife, the dispersion of smaller clearcuts across the forest created by such constraints may have a perverse effect on local environments involving forest fragmentation. One way to approach the problem of fragmentation is to consider habitat area, edge effects, and inter-habitat connections. Integrating these concepts into harvest scheduling problems adds substantial complexity to the models and solution techniques.

We define a *patch* as a continuous area of a particular ecological community surrounded by distinctly different ecological communities, such as a forest patch surrounded by harvest stands or a clearcut surrounded by forest stands. A forest patch can be classified into edge and core area (or interior space). *Core*

[1] Quotation from [Haddad et al., 2015]

area is defined as the interior area of the patch where ecological functioning is not impacted by the effect of immediate surrounding conditions, the so-called *edge effect*. The edge effect corresponds to a buffer area (edge), separating the core area from outside influences, and is due to clearcuts (or patches of early successional species) and non-forest patches.

The core area of a forest patch is determined by the shape, area and immediate surrounding conditions of the patch roughly as follows [Franklin and Forman, 1987, Baskent and Jordan, 1995]:

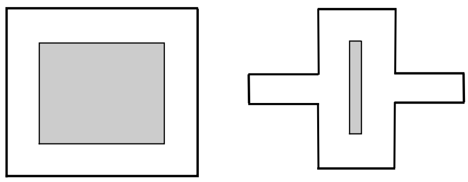

Figure 1: Core area (shaded area) and edge (white area) for two different patch shapes.

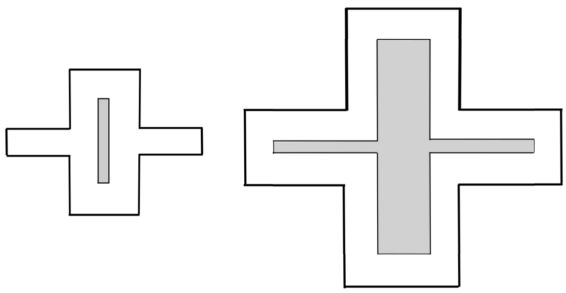

Figure 2: Core area (shaded area) and edge (white area) for two different patch sizes.

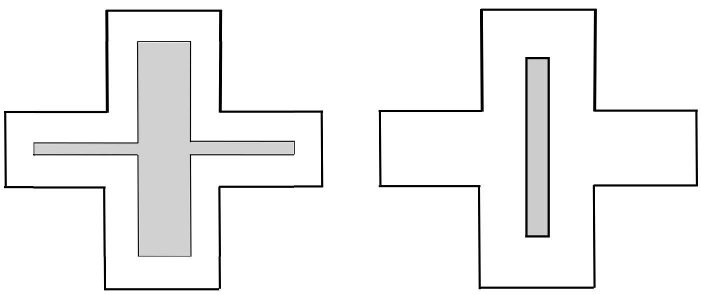

Figure 3: Core area (shaded area) and edge (white area) for two different immediate surrounding conditions.

Shape: Area and immediate surrounding conditions held constant, increasing shape complexity of the patch decreases core area. (Figure 1).

Area: Shape and immediate surrounding conditions held constant, increasing patch area increases core area. (Figure 2).

Immediate surrounding conditions: Similarly, area and shape held constant, augmenting the contrast between the patch and surrounding conditions, increases the distance from the patch boundary inward to a point where edge effects are eliminated, and core area decreases. (Figure 3).

When a habitat is fragmented, core area decreases at the expense of the amount of edge.

Connectivity has been described as the degree to which landscape promotes or prevents species movements among resource patches. It depends on the travel distances that species need to cover to populate the habitats and on the existence of intermediate steps (*e.g.*, forest non-habitat patches) or corridors (*e.g.*, strips of forest) that shorten these distances. As a habitat becomes fragmented, patches become separated from one another (by relatively inhospitable terrain).

Core area can be considered in the models either directly or indirectly. Indirect approaches use patch area and shape, and can be applied to approximate the core area. However, there is no single indicator that summarizes all characteristics of shape. For example, shape index values (values on ratio perimeter to area [McGarigal et al., 2002]), range from one (perfect circle) to higher values (irregular shapes), but two patches of different shapes may share the same shape index value [Baskent and Jordan, 1995]. There are many other approaches that try to grasp the essential characteristics of shapes [Davis, 1977, Moellering and Rayner, 1981, Xia, 1996, Wentz, 2000]. As far as we know, the only indirect approaches to core area used in the literature are the shape index and measures of perimeter and area. Martins et al. [1999], Rebain and McDill [2003a,b], Martins et al. [2005] constrain the minimum total area of forest patches with a minimum area requirement. Tóth et al. [2006] constrain or minimize the total perimeter of the forest patches. All these five works proposed to solve mixed integer linear programming models with constraints on the maximum clearcut area. The approach described in Öhman and Wikström [2008] lies in multi-objective programming, where maximization of the total perimeter of the forest patches is regarded as an additional objective, besides maximizing the net present value. Constraints on clearcut area have not been considered in this study. In order to circumvent computational limitations on the usage of exact methods, some studies proposed heuristics to solve forest planning problems with constraints both on the total area of forest patches above a minimum area requirement and on the clearcut area (simulated annealing and hybrid heuristics by Falcão and Borges [2002], tabu search by Caro et al. [2003]). Öhman and Lämås [2005] used the shape index as a criterion for evaluating the forest fragmentation. A bi-objective problem was solved using simulated annealing, the purpose of which was to maximize the net present value and minimize the shape index of the forest. Constraints on clearcut area have not been considered in this study. Approaches that include core area directly have been developed by, for example, Wei and Hoganson [2007] and Zhang et al. [2011], who solved mixed integer programming formulations, Öhman and Eriksson [1998], Öhman [2000], Öhman et al. [2002], who proposed simulated annealing, Wei and Hoganson [2008] and Hoganson et al. [2005], who proposed dynamic programming-based heuristics, and Neto et al. [2016], who implemented a branch-and-bound approach. From these works, only the last one considered constraints on the clearcut area.

Neto et al. [2013] presented a branch-and-bound approach to solve forest harvest scheduling problems with constraints on the total area of forest patches with a minimum area requirement, as well as on inter-habitat connectivity. This study used the probability of connectivity index [Saura and Hortal, 2007] to measure inter-habitat connections, and included constraints on clearcut area.

Hof et al. [1994] modeled habitat fragmentation indirectly, using mixed integer linear programming formulations that focus on wildlife growth and dispersion as a dynamic and a probabilistic process. Constraints on clearcut area have not been included in the formulations.

The majority of the models applied to forest management problems with forest fragmentation concerns consider a single objective subject to lower or upper bounds on, for example, clearcut size, total perimeter of forest patches, total area of forest patches with a minimum area requirement, total core area, total edge and probability of connectivity index. However, it might be difficult to identify the appropriate threshold requirement bounds that will adequately meet the environmental concerns and, simultaneously, will not be too restrictive. A multi-objective optimization approach may overcome this drawback, minimizing or maximizing some of those measures. In this case, trade-offs between the objectives can be provided, thus allowing the decision maker to choose the one that he/she considers to be the best.

Multi-objective models have been applied to forest management problems, in general, by transforming the multi-objective goals into a weighted-sum single-objective function. These models have been solved by exact methods [Snyder and ReVelle, 1997, Williams, 1998] or heuristics [Öhman and Eriksson, 1998, Öhman, 2000, Öhman et al., 2002, Öhman and Lämås, 2005, Öhman and Wikström, 2008]. Tóth et al. [2006] evaluated the performance of five traditional methods including a new method (the so-called *alpha-delta*) to generate the efficient frontier for a bi-objective harvest scheduling problem, where the objectives are maximization of the net present value and maximization of the total area of forest patches with a minimum area requirement. The traditional methods evaluated in that study are the weighted objective function method, the ϵ-constraining method, the decomposition method based on the Tchebycheff (L_∞) metric, a hybrid method combining the weighted objective function and the ϵ-constraining methods, and the triangles method.

Monte Carlo methods have a long history within numerical algorithms. Monte Carlo integer programming is a heuristic commonly used in forest management problems. This approach was applied, for example, in Nelson and Brodie [1990] to solve a combined harvest scheduling and transportation planning problem with concerns as to clearcut area, and in O'hara et al. [1989], Clements and Dallain [1990] to solve other spatial harvest scheduling problems. Monte Carlo tree search (MCTS) was proposed by **?** for the game of Go (9×9 board) with considerable success. Game-playing is still the area where the algorithm and its many variants are most commonly used. It has also been applied in artificial intelligence approaches to solve problems that can be represented by trees of sequential decisions and planning problems (see Browne et al. [2012] for a comprehensive survey). There are, however, very few publications on its application to solve combinatorial optimization problems. In this field, some results have been provided for general mixed integer optimization, Sabharwal et al. [2012] and Pedroso and Rei [2015] used MCTS to solve specific optimization problems. An application of MCTS to the multi-objective optimization has been performed by Wang and Sebag [2012]. To the best of our knowledge, there are no applications of MCTS to forest management problems.

In this work, we propose a multi-objective MCTS approach to solve a bi-objective harvest scheduling problem with constraints on clearcut area, total habitat area and total core area inside habitats. There are two objectives to be maximized: the net present value of the timber harvested and an index measuring the inter-habitat connectivity. The method is proposed as an approach to generating alternative solutions, indicative of the extent to which economic planning and environmental objectives can be achieved and of the trade-offs between these objectives. MCTS is used as an alternative to standard binary tree search, where the construction and storage of the tree would be computationally expensive, aimed mainly to solve medium and large instances.

We consider mature forests, *i.e.*, forests that are dominated by mature trees, because they are related to the supply of wildlife habitat [Franklin and Forman, 1987]. A tree is mature if it is older than a certain threshold age. Forests are classified into stands, *i.e.*, groupings of vegetation sufficiently uniform in species composition, age, and condition to be managed as single units. We use the term *cluster* to describe a set of contiguous stands or, in other words, a continuous region. A cluster can also be a single stand. A *clearcut* is a cluster of harvested stands surrounded by non-harvested stands (a maximal harvested cluster). It is assumed that a *habitat* is a mature patch (a patch dominated by mature trees) meeting a minimum core area. We assume that this type of patch meets the living needs (*e.g.*, food and shelter) of wildlife. We use the concept of subregions to measure the core area [Öhman and Eriksson, 1998, Wei and Hoganson, 2007, 2008, Zhang et al., 2011], and the *probability of connectivity* index [Saura and Hortal, 2007] to measure the inter-habitat connectivity [Neto et al., 2013].

This paper is outlined as follows. In sections 2 and 3, we describe and formulate the forest planning problem. In Section 4, we explain some underlying concepts of multi-objective optimization. In Section 5, we present the multi-objective Monte Carlo tree search. In Section 6, we report on computational experiments, performed on benchmark instances corresponding to forests ranging from 32 to 1363 stands. In the last section, we present some conclusions.

2 Bi-objective harvest scheduling

The bi-objective harvest scheduling problem (referred to as problem P) deals with determining which stands should be harvested in each period during a given planning horizon in order to maximize both the net present value generated by the harvestings and inter-habitat connectivity. Stand selection is subject to the following non-spatial constraints (volume and average ending age constraints) and spatial constraints (constraints on clearcut size, total habitat area and total core area).

Volume constraints: Lower and upper bounds on the volume of timber harvested in each period. These constraints ensure a regular production of timber.

Average ending age constraints: A minimum average age for the forest at the end of the planning horizon. These constraints aim at mainly preventing harvestings in any part of the forest where any immediate profit can be made.

Maximum clearcut size constraints: An upper bound on the area of each clearcut. For the sake of simplicity, it is considered that stands adjacent to clearcuts cannot just be harvested in the period of the interventions, *i.e.* the minimum number of periods (referred to as *green-up time*) in which stands adjacent to a clearcut cannot be harvested is equal to one period.

Total habitat area constraints: A lower bound on the total area of habitats (edge and core area) in each period.

Total core area constraints: A lower bound on the total core area within habitats in each period.

It is assumed that each stand is harvested at the most once, *i.e.*, that the minimum rotation in the stand is longer than the planning horizon. It is also assumed that harvesting occurs at the beginning of the periods, and that all the periods are of the same length. A minimum age is required for harvesting, and a harvested stand may become mature within the planning horizon.

2.1 Core area and connectivity

We assume that an area in the forest is considered to be core if it is mature and is not impacted on by the effect of clearcutting (edge effect). For the sake of simplicity, it is further assumed that the edge effect induced by a harvested stand a) occurs only during the period of the intervention, b) attains a region — the *impact zone* — starting in the stand's boundary up to a constant distance outward (the depth of the edge effect) [Baskent and Jordan, 1995], c) is the same for all harvested stands, *i.e.* the width of the impact zones is constant.

The intersection of the impact zones of all stands and the stands themselves determines a set of *subregions*. The core area is measured by using these subregions. Each subregion is included in one stand, called the *host* stand, and is defined by a set of stands, the host stand and the so-called *invasive* stands, that is, stands whose impact zones encompass the subregion. Decisions regarding these stands (clearcutting or doing nothing) may determine whether the subregion is a piece of edge or core area. A subregion is neither a core area nor a piece of edge if it is not included in a mature stand. For more details on the characterization of the subregions, see Neto et al. [2016].

The inter-habitat connectivity of the forest is measured by the so-called *probability of connectivity index* [Saura and Hortal, 2007, Neto et al., 2013] for each period t,

$$I_t = \frac{\sum_{h \in \mathcal{H}_t} \sum_{r \in \mathcal{H}_t} \mathsf{s}_h \mathsf{s}_r \mathsf{g}_{hr}}{\mathsf{F}^2}, \tag{1}$$

where: $\mathcal{H}_t$ is the set of all habitats in period t; s_h and s_r are the areas of habitats h and r, with $h, r \in \mathcal{H}_t$; F is the total area of the forest; g_{hr} is the connectivity of the path between h and r with the greatest chance of dispersion for the species considered. g_{hr} assumes values in $[0, 1]$, being one when $h = r$. More details on how to compute g_{hr} can be seen in [Neto et al., 2013].

3 Model

A forest can be be represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where each vertex in $\mathcal{V} = \{1, \ldots, n\}$ corresponds to a stand of the forest, and the vertices of each edge in $\mathcal{E}$ correspond to two adjacent stands. Two different definitions of adjacency are considered [Goycoolea et al., 2005]. For clearcuts, two stands are considered to be adjacent if they share a boundary of positive length. For habitats, they are considered to be adjacent if they share at least one single point [Goycoolea et al., 2009]. The first definition is called *strong adjacency* and the second *weak adjacency*. From an aesthetic standpoint, rooted in beauty and visual appreciation, besides other forest values such as soil erosion and water quality, it is reasonable to assume that a pair of harvested clusters sharing only one single point are non-adjacent and hence belong to two different clearcuts. As for protecting wildlife, with regard to the flow of genetic material throughout the landscape, it is reasonable to assume that a pair of mature clusters sharing only one single point are adjacent and hence belong to the same patch. When strong adjacency is considered, the graph is planar, *i.e.*, it can be drawn on the plane without crossing edges.

A *clique* is a complete subgraph of a graph, *i.e.*, is a subgraph with an edge between each pair of vertices. The clique is maximal if it is not contained in any other clique. Maximal cliques are used in maximum clearcut size constraints. A planar graph has no cliques with more than four vertices. The set of vertices of a connected subgraph corresponds to a cluster.

Example In Figure 4, (b) is a graph representation of the forest in (a) according to strong adjacency. Set $\{B, C, D\}$ is a maximal clique. According to the weak adjacency, stands A and B are adjacent. $\square$

3.1 Notation

Let $\mathcal{T} = \{1, 2, \ldots, T\}$ be the set of T periods in the planning horizon. The following notation is defined.

Indices

t, u - period identifiers;

i, j - stand identifiers;

q - clique identifier;

r - subregion identifier;

c - subset identifier corresponding to a cluster that may be harvested;

h, h' - subset identifiers each one corresponding to a cluster that may be qualified as a habitat.

Parameters

$\mathsf{A}^{\mathsf{max}}$ - maximum clearcut area;

$\mathsf{H}^{\mathsf{min}}$ - minimum habitat area;

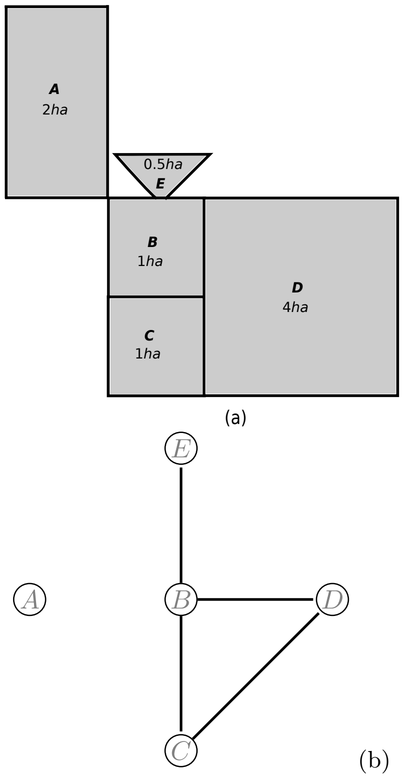

Figure 4: (a) Five-stand mature forest. (b) Graph representation (strong adjacency).

$\mathsf{C}^{\mathsf{min}}$ - minimum core area of a habitat;

$\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$ - minimum total habitat area in each period;

$\mathsf{C}^{\mathsf{min}}_{\mathsf{tot}}$ - minimum total core area in each period;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$ - minimum harvest age, in terms of periods;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ - minimum mature age, in terms of periods;

$\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$ - minimum age requirement, in terms of periods, at the end of the planning horizon;

v_0 - target volume of timber to be harvested in each period.

Sets Sets are defined on the assumption that no intervention is carried out in the forest.

$\mathcal{C}_t$ - set of all possible clusters of stands that are not younger than $\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$ in t, such that the area of each cluster is less than or equal to $\mathsf{A}^{\mathsf{max}}$; the model selects clusters of this type to be harvested in such a way that they become clearcuts (maximal harvested clusters).

$\mathcal{H}'_t$ - set of all possible clusters of stands that are not younger than $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ in t, such that the area of each cluster is not less than $\mathsf{H}^{\mathsf{min}}$; the model selects clusters of this type to remain mature, in such a way that their core areas are no less than $\mathsf{C}^{\mathsf{min}}$;

$\mathcal{V}_t$ - set of stands in period t that are not younger than $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$;

$\mathcal{R}_i$ - set of subregions inside stand i (host stand);

$\mathcal{I}_r$ - set of stands that determine whether subregion r is a piece of edge or a core area;

$\mathcal{K}$ - set of all subsets of vertices of $\mathcal{G}$ that generate maximal cliques.

Data (stand, cluster and subregion attributes)

s_i - area of stand i;

age_{i0} - age, in periods, of stand i in the period before the beginning of the planning horizon;

s_c - area of cluster c;

npv_{ct} - net present value of timber provided by cluster $c \in \mathcal{C}_t$ if it is harvested in period t;

v_{ct} - volume of timber provided by region $c \in \mathcal{C}_t$ if it is harvested in period t;

s_{ir} - area of subregion $r \in \mathcal{R}_i$.

Decision variables

$$z_{ct} = \begin{cases} 1 & \text{if cluster } c \in \mathcal{C}_t \text{ is harvested in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ht} = \begin{cases} 1 & \text{if cluster } h \in \mathcal{H}'_t \text{ is habitat in period } t \\ 0 & \text{otherwise.} \end{cases}$$

$$w_{irt} = \begin{cases} 1 & \text{if subregion } r \in \mathcal{R}_i \text{ is core habitat in period } t \\ 0 & \text{otherwise.} \end{cases}$$

3.2 Mathematical formulation

Problem P can be formulated as follows.

$$\text{maximise} \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}_t} \mathsf{npv}_{ct} z_{ct} \tag{2}$$

$$\text{maximise} \min_{t \in \mathcal{T}} \{I_t\} \tag{3}$$

subject to:

$$I_t = \sum_{h \in \mathcal{H}'_t} \sum_{h' \in \mathcal{H}'_t} \mathsf{g}_{hh'} \mathsf{s}_h y_{ht} \mathsf{s}_{h'} y_{h't} / \mathsf{F}^2, \tag{4}$$

$$\sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}_t:\ i \in c} z_{ct} \leq 1,\ \forall i \in \mathcal{V} \tag{5}$$

$$\sum_{c \in \mathcal{C}_t} \mathsf{v}_{ct} z_{ct} \geq (1 - \alpha) \mathsf{v}_0,\ \forall t \in \mathcal{T} \tag{6}$$

$$\sum_{c \in \mathcal{C}_t} \mathsf{v}_{ct} z_{ct} \leq (1 + \alpha) \mathsf{v}_0,\ \forall t \in \mathcal{T} \tag{7}$$

$$\mathsf{F}\ (T+1) - \sum_{t \in \mathcal{T}} t \sum_{c \in \mathcal{C}_t} \mathsf{s}_c z_{ct} + \sum_{i \in \mathcal{V}} \mathsf{s}_i (1 - \sum_{t \in \mathcal{T}} \sum_{\substack{c \in \mathcal{C}_t: \\ i \in c}} z_{ct}) \mathsf{age}_{i0} \geq \mathsf{F}\ \mathsf{Age}^{\mathsf{min}}_{\mathsf{end}} \tag{8}$$

$$\sum_{c \in \mathcal{C}_t:\ c \cap q \neq \emptyset} z_{ct} \leq 1,\ \forall q \in \mathcal{K},\quad t \in T \tag{9}$$

$$\sum_{u=tmin}^{t} \sum_{c \in \mathcal{C}_u:\ i \in c} z_{cu} + \sum_{h \in \mathcal{H}'_t:\ i \in h} y_{ht} \leq 1,\ \forall t \in \mathcal{T}, i \in \mathcal{V}_t \tag{10}$$

$$w_{irt} + \sum_{\substack{c \in \mathcal{C}_t: \\ j \in c}} z_{ct} \leq 1,\ \forall t \in \mathcal{T}, i \in \mathcal{V}_t, r \in \mathcal{R}_i, j \in \mathcal{I}_r \setminus \{i\} \tag{11}$$

$$w_{irt} \leq \sum_{h \in \mathcal{H}'_t:\ i \in h} y_{ht},\ \forall t \in \mathcal{T}, i \in \mathcal{V}_t, r \in \mathcal{R}_i \tag{12}$$

$$\sum_{i \in h} \sum_{r \in \mathcal{R}_i} \mathsf{s}_{ir} w_{irt} \geq \mathsf{C}^{\mathsf{min}} y_{ht},\ \forall t \in \mathcal{T}, h \in \mathcal{H}'_t \tag{13}$$

$$\sum_{h \in \mathcal{H}'_t} \mathsf{s}_h y_{ht} \geq \mathsf{H}^{\mathsf{min}}_{\mathsf{tot}},\ \forall t \in \mathcal{T} \tag{14}$$

$$\sum_{i \in \mathcal{V}_t} \sum_{r \in \mathcal{R}_i} \mathsf{s}_{ir} w_{irt} \geq \mathsf{C}^{\mathsf{min}}_{\mathsf{tot}},\ \forall t \in \mathcal{T} \tag{15}$$

$$z_{ct} \in \{0, 1\},\ \forall t \in \mathcal{T}, c \in \mathcal{C}_t \tag{16}$$

$$y_{ht} \in \{0,1\},\ \forall t \in \mathcal{T}, h \in \mathcal{H}'_t \tag{17}$$

$$w_{irt} \in \{0,1\},\ \forall t \in \mathcal{T}, i \in \mathcal{V}_t, r \in \mathcal{R}_i, \tag{18}$$

with $tmin = \max\{1, t - (\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}} - 1)\}$ (constraints (10)). Here, $[tmin, t]$ is the time interval (in periods) for a stand to be mature in period t.

Expression (2) specifies the first objective of the problem, namely the maximization of the net present value of timber harvested.

Expression (3) indicates the second objective, which is the maximization of the minimum value for the connectivity index over all periods.

Constraints (5) ensure that each stand is harvested at the most once in the planning horizon. Constraints (6) and (7) require minimum and maximum volumes of timber to be harvested in each period, respectively. Constraint (8) requires the weighted average age of the forest at the end of the planning horizon, in period $T+1$, to be at least $\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$ periods.

Constraints (9), the so-called clique constraints, guarantee that if a cluster is harvested in a certain period then no adjacent clusters (clusters intersecting the same clique) are harvested in this period. This prevents the formation of clearcuts whose areas are greater than $\mathsf{A}^{\mathsf{max}}$, because the area of each cluster is already less than or equal to $\mathsf{A}^{\mathsf{max}}$. Constraints (10) ensure that each stand belongs at the most to one cluster selected either to be harvested or to be a habitat. If a stand is selected to belong to a habitat in period t, it cannot be selected for harvesting from period $tmin$ to t, as a harvested stand will be mature after $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ periods (and harvestings occur at the beginning of the periods).

Constraints (11) and (12) are related to the definition of the core area. For each period, constraints (11) ensure that a mature subregion is not a core area when an invasive stand is harvested. If $\sum_{c \in \mathcal{C}_t : j \in c} z_{ct} = 1$ (a cluster containing j is harvested) then, by constraints (11), $w_{irt} = 0$ (r is not a core area in t), where r is a mature subregion and j is an invasive stand of r. Constraints (12) guarantee that a mature subregion is not a core area if there are no habitats that have the stand where the subregion is. If $\sum_{h \in \mathcal{H}'_t :\ i \in h} y_{ht} = 0$ (no clusters containing i are selected to be habitat) then, by constraints (12), $w_{irt} = 0$ (r is not a core area in t), where r is a mature subregion and i is the host stand of r. Constraints (13) are related to the definition of a habitat, thus ensuring the minimum requirement of the core area of a habitat. The left-hand side of constraint (13) for cluster $h \in \mathcal{H}'_t$ represents the core area of h. If h is selected as habitat ($y_{ht} = 1$) then its core area cannot be less than $\mathsf{C}^{\mathsf{min}}$. Constraints (14) and (15) require a minimum value for the total habitat and core areas in each period, respectively.

Constraints (16), (17) and (18) state the binary nature of the decision variables.

The main drawback of model (2)-(18) is the number of variables, which grows exponentially with the number of stands. This number also grows with greater values of $\mathsf{A}^{\mathsf{max}}$ (variables z), with smaller values of $\mathsf{H}^{\mathsf{min}}$ (variables y), and with larger impact zone widths (variables w). In [Neto et al., 2016], a commercial mixed integer programming solver was not able to find a feasible solution of the single objective model (2), (5)-(18), within two hours, for any of the instances used in the current paper except the smallest ones (with 32 and 72 stands). These computational results make practically useless any linearization of the objective function (3) in order to solve model (2)-(18) through a general purpose solver.

3.3 Relationship between probability of connectivity index and habitat area

Larger values of the connectivity index enhances not only the connectivity between habitats (by reducing the inter-habitat distances) but also the area of all habitats (by increasing the number of habitats or the area of each habitat). We now show that if the connectivity index I_t is greater than or equal to $(H_{\text{tot}}^{\text{min}}/\mathsf{F})^2$, there is no need to check constraint (14) for period t on the total habitat area.

Proposition 3.1. *Let $I_t \geq \left(\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}/\mathsf{F}\right)^2$ for some period $t \in \mathcal{T}$. Then, constraint (14) for period t is satisfied.*

Proof. Given a period t, let $y_{ht} = 1$ for all habitats h in $\mathcal{H}_t$ (set of all habitats in t) and $y_{ht} = 0$ for $h \in \mathcal{H}'_t \setminus \mathcal{H}_t$. Consider that $I_t \geq \left(\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}/\mathsf{F}\right)^2$.

The definition of I_t and the instantiation for y assure that

$$\sum_{h\in\mathcal{H}_t}\sum_{h'\in\mathcal{H}_t} \mathsf{s}_h \mathsf{s}_{h'} \mathsf{g}_{hh'} \geq (\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}})^2.$$

Considering that the values of $\mathsf{g}_{hh'}$ are bounded above by 1 and $\mathsf{g}_{hh} = 1$, then

$$\sum_{h\in\mathcal{H}_t} {\mathsf{s}_h}^2 + \sum_{h\in\mathcal{H}_t}\sum_{h'\in\mathcal{H}_t:h'\neq h} \mathsf{s}_h \mathsf{s}_{h'} \geq (\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}})^2.$$

The left-hand side of this expression is the square of the sum $\sum_{h\in\mathcal{H}_t} \mathsf{s}_h$. Therefore,

$$\left(\sum_{h\in\mathcal{H}_t} \mathsf{s}_h\right)^2 \geq (\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}})^2 \Leftrightarrow \sum_{h\in\mathcal{H}_t} \mathsf{s}_h \geq \mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}.$$

That is, constraint (14) for period t is satisfied. □

4 Used concepts in multi-objective optimization

A multi-objective optimization problem can be described as

$$\max_{x\in X} \; f(x) = (f_1(x), f_2(x), \ldots, f_p(x)),$$

where $p > 1$ is the number of objectives and X denotes the set of feasible solutions. It is assumed that X is compact (closed and bounded) and non-empty.

In multi-objective optimization, there does not typically exist a feasible solution that maximizes all objective functions simultaneously. Therefore, attention is paid to solutions that cannot be improved in any of the objectives without degrading at least one of the other objectives.

A solution $x \in X$ is *efficient* or *Pareto-optimal* if and only if there is no other solution $x' \in X$ such that $f_i(x') > f_i(x)$ for at least one of the objectives and $f_i(x') \geq f_i(x)$ for the others, that is, any solution x' that improves at least one of the objectives, also degrades the others. A solution $x \in X$ is *weakly efficient* if and only if there is no other solution $x' \in X$ such that $f_i(x') > f_i(x)$ for all objectives, that is, any solution x' that improves at least one of the objectives, also maintains or degrades the others. Thus, if x is Pareto-optimal then it is also weakly efficient, but the reverse may not be true.

Let $f(X)$ be the image of X in the objective space. The output of a Pareto-optimal solution is called *non-dominated* or *Pareto point*. The output of a weakly efficient solution is called *weakly non-dominated point*. Thus, a Pareto point is also weakly non-dominated, but the reverse may not be true. The outputs of the other feasible solutions are called *dominated points*. The set of all Pareto points is called the *Pareto front* or Pareto frontier. So, the Pareto front is made up of all points that correspond to solutions that cannot be improved in any of the objectives without degrading at least one of the other objectives.

When the feasible region X is non-convex (*e.g.*, when problems have integer variables), there are two types of Pareto points: supported and unsupported ones. Consider two elements y and y' in the objective space. Element y dominates y', which is denoted as $y \succeq y'$, if and only if the components of y are greater than or equal to the corresponding components of y'. A Pareto point y is called *unsupported* if it is dominated by a convex combination of Pareto points; if there are no convex combinations of Pareto points that dominate y, y is a *supported* point.

The Pareto front of a multi-objective optimization problem is bounded by the so-called *nadir* objective vector $\mathsf{Z}^{\mathsf{nad}}$ and an *ideal* objective vector $\mathsf{Z}^{\mathsf{ideal}}$. The components of a nadir and an ideal objective vectors define lower and upper bounds on the objective function values of Pareto-optimal solutions, respectively. The nadir objective vector is determined as

$$\mathsf{Z}_i^{\mathsf{nad}} = \inf_{x \in X \text{ is Pareto-optimal}} f_i(x) \text{ for all } i = 1, \dots, p$$

and the ideal objective vector as

$$\mathsf{Z}_i^{\mathsf{ideal}} = \sup_{x \in X} f_i(x) \text{ for all } i = 1, \dots, p.$$

In practice, these vectors may be approximated, as typically the whole Pareto-optimal set is unknown.

The main purpose of multi-objective optimization is to find the Pareto front for a given instance. A good multi-objective heuristic optimizer is one that provides a good approximation of the Pareto front, that is, a representative subset of the front that does not contain certain points (*e.g.*, unsupported Pareto points).

Many metrics have been proposed to compare the quality of different approximation sets of the Pareto front, by mapping an approximation set to a real value. Perhaps the most popular is the *hypervolume indicator*, also called the S-metric or Lebesgue measure. The hypervolume measure was originally proposed by Zitzler and Thiele [1998], who called it the size of the dominated space Zitzler [1999]. It has been used in comparative studies [Zitzler et al., 2003], and it has also been integrated into heuristics, such as evolutionary multi-objective algorithms [Zitzler and Thiele, 1998, Deb et al., 2003, Fleischer, 2003] and Monte Carlo tree search [Wang and Sebag, 2012].

The hypervolume encapsulates in a single value a measure of the spread of Pareto points along the Pareto front. Additionally, it clearly reflects well a number of important quality criteria, such as the proximity of the set of dominated points to the Pareto front. The hypervolume indicator $V(F)$ of a given set F of Pareto points is defined as the volume of the objective space dominated by F and bounded below by a reference point r - a point that should be dominated by all Pareto points (usually, a nadir point). More formally,

$$V(F) = \mu(\{f(x') : f(x) \succeq f(x') \succeq r,\ x \in X \text{ is Pareto-optimal}, x' \in X\}),$$

where μ is the Lebesgue measure (corresponding to the shadowed area in Figure 5).

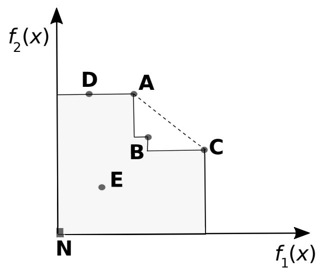

Figure 5: Example of a hypervolume in a Pareto front of a bi-objective optimization problem. A, B, C are Pareto points; A and C are supported; B is unsupported (it is not dominated by points A and C individually, but it is dominated by some convex combinations of A and C); D is a weakly non-dominated point. E and N are dominated points; N is a nadir point. The hypervolume of $\{A$, B, C$\}$ is given by the shaded area.

5 Monte Carlo Tree Search

5.1 General description

Monte Carlo tree search (MCTS) is a method for exploring a search tree and exploiting its most promising regions. The tree is built in an incremental and asymmetric manner using stochastic simulations. Each iteration of the method consists of four main steps: *node selection*, *expansion*, *simulation* and *backpropagation*. An iteration starts from the root node or initial state and recursively selects a node (node selection step) until reaching a node in the tree which is not fully expanded (leaf node). Once at a leaf node, MCTS creates one or more new nodes (expansion step), and for each child node it applies a simulation (simulation step) until a solution is reached (terminal state). Finally, statistics kept at each node in the path are updated, until reaching the root node (backpropagation step). This process is repeated until a termination criterion is met (such as reaching a number of iterations or an elapsed time). A great benefit of MCTS is that the values of intermediate states do not have to be evaluated: only the value of the terminal state at the end of each simulation is required.

Each step of MCTS is explained in more detail below.

Selection: Starts from the root node and iteratively selects the child node which currently looks more promising, until a node which has not yet been fully expanded is reached. The best known version of MCTS is the Upper Confidence Bounds for Trees (UCT), first introduced by Kocsis and Szepesvári [2006], which employs equation (19) (referred to as UCB1) as a tree node selection policy. This policy balances the exploitation of known moves which appear to be promising (first term of (19)) and the exploration of a new portion of the search space (second term):

$$U(k) = Q(k) + \sigma\sqrt{\frac{\ln s_{p(k)}}{s_k}}, \tag{19}$$

where σ is a constant that balances both terms (the value depends on the problem approached, but it is common to find $\sigma = \sqrt{2}$), $s_{p(k)}$ is the number of simulations performed under the parent node $p(k)$, and s_k is the number of simulations performed under the child node k (*i.e.*, simulations started from k or from any node in the subtree under k). At each node, the child with maximum $U(k)$ is selected, until an unexpanded node is reached.

In game-playing, $Q(k)$ is typically taken to be the average reward of simulations run from k. In applying MCTS to solve optimization problems there are significant differences in relation to its application in game-playing. An important difference concerns the evaluation of nodes and their associated statistics. Whereas in game-playing a branch with a high average win rate is suggestive of a strong line of play, in optimization, the average rate under a node is not a good estimator of the best solution to the node's underlying subproblem. Additionally, rewards in game playing often take 0 and 1 values for loss and win, respectively; objective functions, on the other hand, may take arbitrary values. Since (19) was designed with rewards in the $[0, 1]$ interval, in order to maintain the proper balance between the two components of this equation, Pedroso and Rei [2015] propose the following shape for function $Q(k)$:

$$Q(k) = \frac{\widehat{w}^* - \widehat{z}_k}{\widehat{w}^* - \widehat{z}^*}, \tag{20}$$

where $\widehat{z}^*$ and $\widehat{w}^*$ are, respectively, the best and the worst simulation results found in the part of the tree explored so far, and $\widehat{z}_k$ is the best simulation outcome under the node.

Expansion: A step that adds nodes to the MCTS tree. One or more child nodes of the selected node k are added to expand the tree, according to the available actions. Two strategies for node expansion could be considered [Pedroso and Rei, 2015]. In the *single expansion*, a single child node is created using a randomly chosen unexplored decision in k; other unexplored decisions are kept for a later time when node k is again selected for expansion. In *full expansion*, all children of node k are immediately created by generating all possible decisions in the node.

Simulation: From each node created in the expansion step, a simulation is performed, until a solution is reached. Various approaches can be applied. The simplest approach consists in taking uniform random decisions, requiring nothing more than a generative model of the problem. Heuristic construction algorithms incorporate domain-specific knowledge, and typically allow for faster convergence at the expense of simplicity and generality.

Backpropagation: Step that propagates the outcome of the simulation backwards from the selected node to the root. This updates statistics (simulation scores and counts) that inform future tree policy decisions on all nodes between the selected node and the root.

As opposed to metaheuristics, MCTS is not based on neighborhoods, and hence does not suffer from local optima. However, for medium or large instance sizes, the search tree cannot be fully explored, and hence there is not guaranty that the solution is optimal, unless the lower and the upper bounds overlap.

5.2 MCTS for multi-objective optimization

In our problem, at each node k in the MCTS the solution x is an integer vector with n components, where x_i is the period at which stand i is harvested. If the stand is not harvested across the planning horizon, then $x_i = 0$. Observe that solution x has a correspondence with variables z_{ct}, as $x_i = \sum_{t \in \mathcal{T}} \sum_{c \in \mathcal{C}_t : i \in c} t z_{ct}$.

In the expansion step, two child nodes are immediately created (full expansion), corresponding to the decision of harvesting stand i or not in period t. The admissibility of the solution corresponding to each children is verified. Let $\mathcal{S}$ be the set of all pairs (stand i, period t) such that i is available for harvesting in t (i is not younger than $\mathsf{Age}_{\mathsf{cut}}^{\mathsf{min}}$ in t, and its area is not greater than $\mathsf{A}^{\mathsf{max}}$), sorted by descending order of the net present value. The tuple (i, t) chosen for expansion is the first element of the ordered set $\mathcal{S}$. In the simulation step, a child is generated corresponding to the decision of harvesting stand i or not in period t ; in this case, the tuple (i, t) is chosen randomly. The decisions to harvest or not are available, and depend on the admissibility of the corresponding solution; but if both decisions correspond to feasible solutions, selection is random. After a tuple is fixed, it is removed from $\mathcal{S}$. A terminal state is reached at the end of a simulation, when all (stand, period) pairs are fixed.

Moving from single to multi-objective MCTS (MO-MCTS) requires some modifications in the baseline algorithm. Firstly, MO-MCTS maintains a set F of all non-dominated solutions found in terminal states. Secondly, the result of the simulation, Z, is now a vector of results for each objective. Let $Z = (npv, I)$, where npv is the net present value and I is the value of the connectivity index corresponding to solution x. So as to use equation (20), we adapted an approach based on the value of the hypervolume [Wang and Sebag, 2012]. If Z is non-dominated in F, the value of Z is replaced by $V(F \cup \{Z\})$; in our case, for dominated solutions, Z is replaced by $V(F)$ minus the distance from Z to the convex envelope of F. The main steps of MO-MCTS are presented in Algorithm 1.

Even though we are using only two objectives, the method based on hypervolume described in this paper can be employed with any number of objectives [Zitzler and Thiele, 1998, Wang and Sebag, 2012].

With MCTS, the solution at a child node is evaluated by taking the solution at the parent node and considering the changes corresponding to the stand in which the decision is taken. This evaluation procedure may provide a considerable speedup of the method.

Algorithm 1: Main steps of Multi-objective Monte Carlo Tree Search

Step 1 ***Initialization:***
set $F := \emptyset$
create root node s
Step 2 ***Termination:***
if *time limit is reached* **then**
  if $F = \emptyset$ **then** nothing can be said
  else propose heuristic solutions that yielded F
Step 3 ***Node selection:***
repeat
  starting from s, recursively select child k with maximum $U(k)$
until k *is a leaf node*
Step 4 ***Expansion:*** let $C := \{k+1, k+2\}$ be the set of children obtained from expanding k
foreach $k' \in C$ **do**
Step 5   ***Simulation:*** let Z be the result of a simulation from k'
Step 6   ***Backpropagation:*** propagate Z up until reaching s
  if Z *is not dominated by any point in* F **then**
    $F := F \setminus \{S \in F : S \text{ is dominated by } Z\}$
    $F := F \cup \{Z\}$
go to Step 2

6 Computational experiment

6.1 Instances

In order to test the performance of the Monte Carlo tree method we have used instances publicly available in the Internet[2]. The instances include: *El Dorado*, a National Forest in northern California, USA (referred to in Goycoolea et al. [2005, 2009], Könnyű and Tóth [2013]); *Stafford*, a forest in British Columbia, Canada ([Crowe et al., 2003]); *Kittaning*4, *Bear Town*, *PhyllisLeeper* and *Five-Points*, forests in Pennsylvania, USA ([Könnyű and Tóth, 2013]); *WLC* ([Bettinger et al., 2002]); *FLG*9 and *FLG*10 ([Paradis and Richards, 2001]), the computer generated instances (through the Forest Landscape Generator).

The number of stands ranges from 32 (Kittaning4) to 1363 (El Dorado) and the number of edges from 48 to 4087 (considering weak adjacency). The length of the temporal horizon ranges from 3 to 8 periods. Some instances are tested for different lengths of horizon T; the notation used is $instance_T$. Table 1 summarizes the characteristics of the instances.

Values for α, v_0, $\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$, $\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$, $\mathsf{C}^{\mathsf{min}}_{\mathsf{tot}}$ and other parameters of the formulation (2)-(18) are fixed as in Neto et al. [2013] and Neto et al. [2016] (see Appendix A for a summary description). Table 2 shows the values of the parameters $\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$, $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$, $A^{\max}$, $\mathsf{C}^{\mathsf{min}}$, $\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$, $\mathsf{C}^{\mathsf{min}}_{\mathsf{tot}}$ (as a percentage of the total area of the forest F) and v_0.

An impact zone width of $50m$ has been used to generate the subregions. Table 3 describes the characteristics of the subregions according to this impact

[2]Instances have been obtained at the web site `http://www.unbf.ca/fmos/`

Instance	No. stands	Weak adjacency No. edges	Strong adjacency No. edges	Strong adjacency No. cliques	No. periods (years per period)	$\overline{\mathsf{age}}_1$ (years)	F (ha)
El Dorado	1363	4087	3617	2041	3 (10)	105.86	21147
Stafford	1008	2113	2066	1163	3 (10)	50.83	10444
FLG9$_{3/8}$	850	2524	2388	1420	3 / 8 (5)	31.91	10000
FLG10$_{3/8}$	763	2262	2137	1269	3 / 8 (5)	27.02	10000
Five Points$_{3/5}$	90	164	149	88	3 / 5 (10)	63.11	677
PhyllisLeeper$_{3/5}$	89	161	131	86	3 / 5 (10)	94.38	646
WLC$_{3/7}$	73	114	98	63	3 / 7 (5)	46.58	897
Bear Town$_{3/5}$	71	148	101	64	3 / 5 (10)	95.49	546
Kittaning4$_{3/5}$	32	48	47	25	3 / 5 (10)	66.59	238

Table 1: Characteristics of the instances; $\overline{\mathsf{age}}_1$ is the average age of the stands in the first period and F is the area of the forest.

Instance	$\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$ (periods/years)	$\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ (periods/years)	$\mathsf{A}^{\mathsf{max}}$ (ha)	$\mathsf{C}^{\mathsf{min}}$ (ha)	$\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$ (% of F)	$\mathsf{C}^{\mathsf{min}}_{\mathsf{tot}}$ (% of F)	v_0 (m^3)
El Dorado	6/60	8/80	40	40	10	5	683632
Stafford	6/60	6/60	40	40	10	7.5	668558
FLG9$_{3/8}$	8/40	8/40	46	46	5	2.5	114149 / 76099
FLG10$_{3/8}$	8/40	8/40	46	46	5	2.5	102638 / 68425
Five Points$_{3/5}$	6/60	6/60	40	40	10	5	426 / 256
Phyllis Leeper$_{3/5}$	8/80	8/80	40	40	10	5	4437 / 3169
WLC$_{3/7}$	8/40	8/40	40	40	10	4.5	18415 / 10523
Bear Town$_{3/5}$	8/80	8/80	40	40	10	5	3468 / 2890
Kittaning4$_{3/5}$	6/60	6/60	20	20	15	10	695 / 417

Table 2: Values of the parameters $\mathsf{Age}^{\mathsf{min}}_{\mathsf{cut}}$ (minimum harvest age), $\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}}$ (minimum mature age), $\mathsf{A}^{\mathsf{max}}$ (maximum clearcut area), $\mathsf{C}^{\mathsf{min}}$ (minimum core area for a habitat), $\mathsf{H}^{\mathsf{min}}_{\mathsf{tot}}$ (minimum total habitat area in each period), $\mathsf{C}^{\mathsf{min}}_{\mathsf{tot}}$ (minimum total core area in each period) and v_0 (target volume for each period).

zone width.

Instance	Stands		Subregions		
	No.	Average area (ha)	No.	Average area (ha)	Average no. stands
El Dorado	1363	15.52	19910	1.06	2.00
Stafford	1008	10.36	10560	0.99	2.58
$\mathrm{FLG9}_{3/8}$	850	11.76	10151	0.99	2.47
$\mathrm{FLG10}_{3/8}$	763	13.10	9129	1.10	2.47
Five $\mathrm{Points}_{3/5}$	90	7.52	1347	0.50	3.00
Phyllis $\mathrm{Leeper}_{3/5}$	89	7.26	961	0.67	2.70
$\mathrm{WLC}_{3/7}$	73	12.28	675	1.33	2.66
Bear $\mathrm{Town}_{3/5}$	71	7.69	776	0.70	2.82
$\mathrm{Kittaning4}_{3/5}$	32	7.44	337	0.71	2.70

Table 3: Description of the forests in terms of stands and subregions according to an impact zone width of $50m$; with respect to subregions, the third column displays the average number of stands that determine whether a subregion is a piece of edge or core area ($\sum_{i\in\mathcal{V}}\sum_{r\in\mathcal{R}_i}|\mathcal{I}_r|/\sum_{i\in V}|\mathcal{R}_i|$).

6.2 Computational Results

The platform used was an Intel Core i7 quad-core CPU, running at 3.4 GHz in a Mac OS X version 10.10.2, with 24 GB of RAM. All programs were implemented in Python (version 2.7.9). We ran MO-MCTS 12 times for each instance. MO-MCTS was allowed to run for two hours at each time.

For each instance, a solution that appeared to be efficient within a run may cease to be efficient among all runs. We shall refer to the first solution as *false-efficient* and the solution that remain efficient as *keep-efficient*. Table 4 displays, for each instance, the average number of efficient solutions (including false-efficient and keep-efficient), keep-efficient solutions and simulations obtained by each run. The average number of efficient solutions ranged from 2.50 (El Dorado) to 13.58 ($\mathrm{Kittaning4}_3$). $\mathrm{Kittaning4}_3$ was the instance with the largest number of simulations, as opposed to El Dorado. The smallest numbers of efficient solutions and simulations were observed in case of the large instances. For the medium sized instance WLC, a small number of efficient solutions were provided within a large number of simulations. We noted a tendency for many efficient solutions to cease to be efficient among all runs. This leads us to conclude that MO-MCTS must be run several times to find better approximations to the optimal Pareto front.

Figures 6, 7 and 8 represent the Pareto fronts obtained for $\mathrm{Kittaning4}_3$, $\mathrm{PhyllisLeeper}_3$ and El Dorado, respectively. Each graphic corresponds to one run (at the top, the first six runs), where the x-axis and y-axis represent the values of npv ($\times 10^3$ euros) and I, respectively. The dark solid line corresponds to the best Pareto front for each run (false-efficient and keep-efficient solutions) and the dashed line corresponds to the best Pareto front over all runs (keep-efficient solutions). Gray lines show the evolution of the Pareto front throughout each run. This representation enables us to observe, in each figure, the sequence of non-dominated points generated by an MO-MCTS run. It also shows that each run generated different sets of efficient solutions, hence producing a different Pareto front approximation.

Instance	Average number of efficient solutions	keep-efficient solutions	simulations
El Dorado	2.50	0.67	9172.83
Stafford	3.50	0.42	16131.92
$FLG9_3$	3.75	0.33	30022.75
$FLG9_8$	2.58	0.42	14015.75
$FLG10_3$	7.00	0.67	43826.42
$FLG10_8$	2.42	0.42	18312.42
$FivePoints_3$	10.42	0.25	3326018.58
$FivePoints_5$	10.50	1.33	2033565.67
$PhyllisLeeper_3$	10.17	0.83	4416487.42
$PhyllisLeeper_5$	9.00	0.25	3180922.75
WLC_3	3.91	0.33	6590686.27
WLC_7	2.92	0.33	3731875.00
$BearTown_3$	10.17	1.00	7326169.33
$BearTown_5$	11.00	1.17	5572913.08
$Kittaning4_3$	13.58	1.17	14763647.25
$Kittaning4_5$	12.67	1.50	9459201.92

Table 4: Average number of efficient solutions (false-efficient and keep-efficient), keep-efficient solutions and simulations by run; a solution that appeared to be efficient that ceases/continues to be efficient among all runs is false-efficient/keep-efficient.

These figures and similar figures for the other instances allow us to observe, with respect to the trade-off between net present value of timber harvested and probability of connectivity index, that ensuring a more inter-habitat connectivity is not always obtained at the expense of a reduction in the net present value.

To compare instances with respect to the variation in the hypervolumes over the runs, we use the dimensionless coefficient of variation (DC_V) and relative range (R_V). Both are measures of relative dispersion of that variable and they compare a dispersion measure with a localization measure. The coefficient of variation is the ratio between the standard deviation of the hypervolumes and the average of these values, and compares standard deviation with average. The relative range is the ratio between the total range of the hypervolumes (the maximum minus the minimum) and the maximum hypervolume, and compares the total range with the maximum value. The values for DC_V and R_V are displayed in table 5. For DC_V, the smallest value occurs for WLC_3 (0.0005) and the largest value for $FLG9_8$ (0.3214). Concerning R_V, the smallest value occurs also for WLC_3 (0.0016) and the largest value for $FivePoints_5$ (0.4426). Since both indicators assume a wide range of values, the approach appears to provide from very small variability to considerable variability on the hypervolume over the runs. Nevertheless, fourteen instances (87.5% of the instances) and ten instances (62.5% of the instances) have values of DC_V and R_V, respectively, smaller than half of the largest values of DC_V and R_V observed.

Table 6 shows some characteristics of the keep-efficient solutions with the best net present value or the best connectivity index. The values of the number, total area and total core area of habitats refer to the last period of the planning horizon. There is a tendency for the total area and total core area of habitats to increase in the solutions with the best I, in particular, for the small and medium instances. For these instances, generally the number of habitats remained the same. For $Kittaning4_3$ and $FivePoints_5$, this number increased from one to two

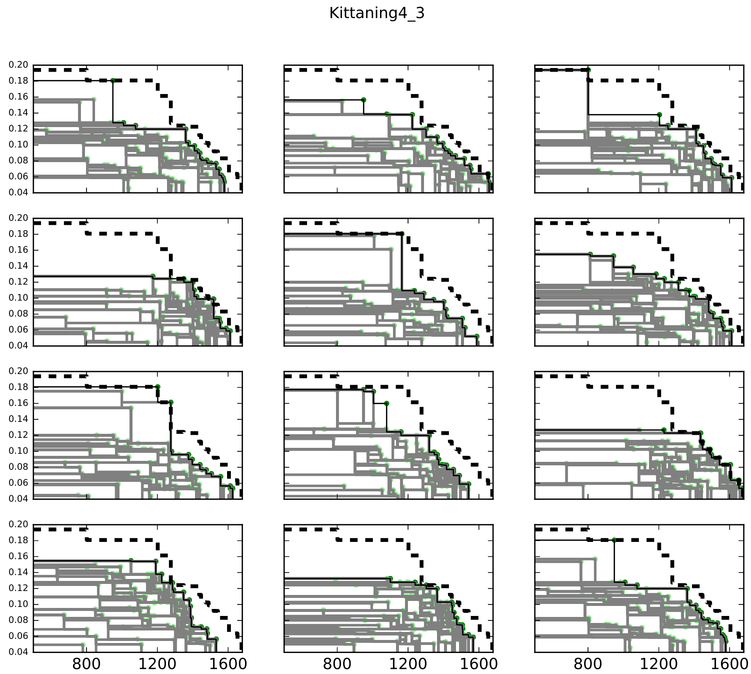

Figure 6: Pareto fronts for Kittaning4_3 (x-axis and y-axis represent the values of npv ($\times 10^3$ euros) and I, respectively). The gray lines show the evolution of the Pareto front throughout each run; the dark solid line is the best Pareto front (false-efficient and keep-efficient solutions); the dotted line represents the best Pareto front over all runs (keep-efficient solutions).

Instance	Best net present value (euros)	Best connectivity index	DC_V	R_V
El Dorado	2020748	0.0429	0.0899	0.2869
Stafford	134494025	0.0042	0.0410	0.1343
FLG9_3	44808615	0.0121	0.0277	0.0081
FLG9_8	50346461	0.0117	0.3214	0.1652
FLG10_3	40675587	0.0113	0.0310	0.1114
FLG10_8	46257990	0.0110	0.0292	0.0852
FivePoints$_3$	536853	0.0527	0.0559	0.2028
FivePoints$_5$	393334	0.0796	0.2565	0.4426
PhyllisLeeper$_3$	5589066	0.0845	0.1265	0.3485
PhyllisLeeper$_5$	4949207	0.0439	0.1084	0.2970
WLC$_3$	4635768	0.0174	0.0005	0.0016
WLC$_7$	4180389	0.0174	0.0018	0.0047
BearTown$_3$	4368877	0.0965	0.0590	0.1726
BearTown$_5$	4510859	0.0561	0.0233	0.0800
Kittaning4_3	1675397	0.1940	0.0983	0.2393
Kittaning4_5	13388788	0.2567	0.1498	0.3427

Table 5: Best net present value and best connectivity index over the keep-efficient solutions. Coefficient of variation (DC_V) and relative range (R_V) of the hypervolume over all runs.

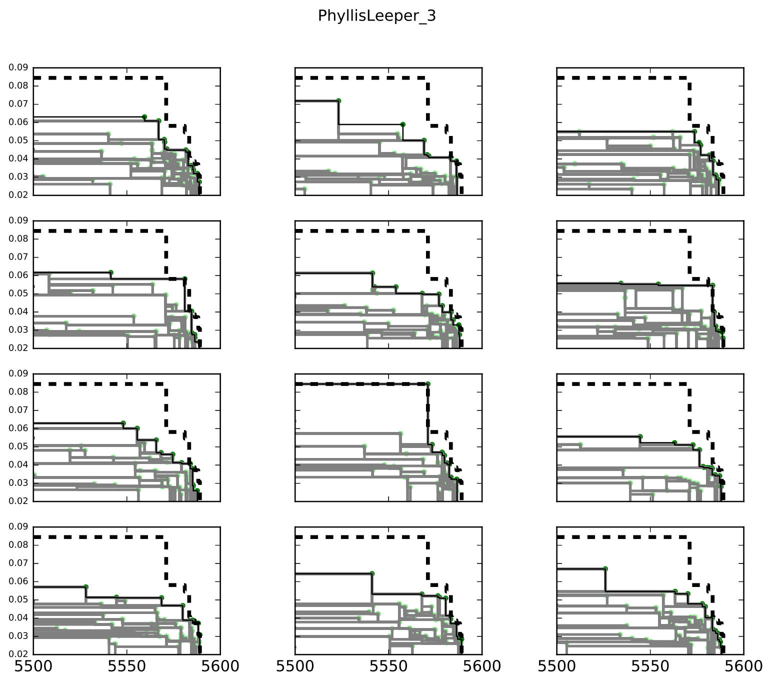

Figure 7: Pareto fronts for PhyllisLeeper$_3$ (x-axis and y-axis represent the values of npv ($\times 10^3$ euros) and I, respectively). The gray lines show the evolution of the Pareto front throughout each run; the dark solid line is the best Pareto front (false-efficient and keep-efficient solutions); the dotted line represents the best Pareto front over all runs (keep-efficient solutions).

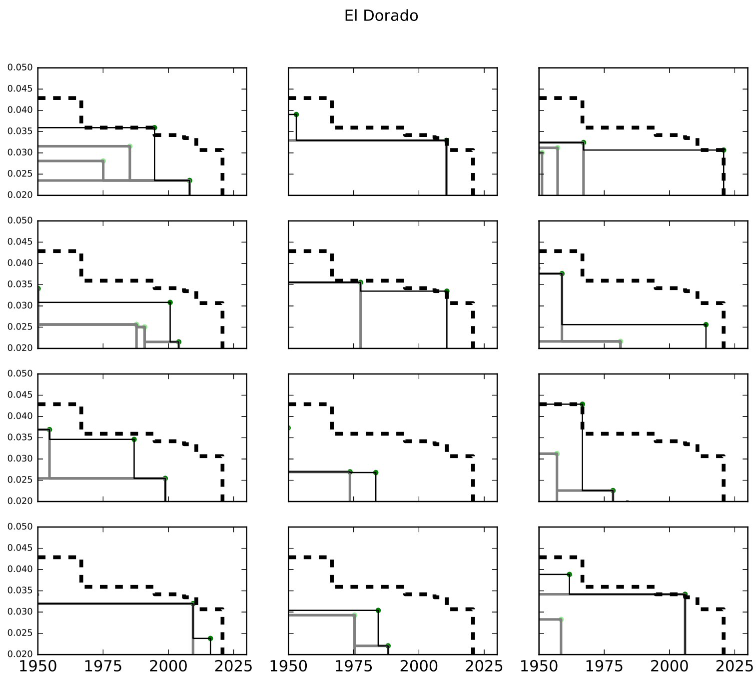

Figure 8: Pareto fronts for El Dorado (x-axis and y-axis represent the values of npv ($\times 10^3$ euros) and I, respectively). The gray lines show the evolution of the Pareto front throughout each run; the dark solid line is the best Pareto front (false-efficient and keep-efficient solutions); the dotted line represents the best Pareto front over all runs (keep-efficient solutions).

habitats (Figure 9). For almost all the large instances, the number of habitats decreased. For $FLG10_8$, the number of habitats did not change, and for $FLG9_3$, the number of habitats decreased (significantly) as well as their total area and core area (slightly). Figure 10 shows that these habitats become closer to each other compared with those solutions that provide the best npv.

Instance	Best net present value						Best connectivity index					
	npv (euros)	I	time (s)	No. habitats	Habitat area (ha)	Core area (ha)	npv (euros)	I	time (s)	No. habitats	Habitat area (ha)	Core area (ha)
El Dorado	2020748	0.0307	7200	11	5145	4388	1966640	0.0429	7200	9	5651	4908
Stafford	134494025	0.0035	1644	17	1045	983	127204397	0.0042	5173	14	1049	965
$FLG9_3$	44808615	0.0099	4382	18	1502	1462	42661754	0.0121	4288	9	1501	1431
$FLG9_8$	50346461	0.0097	4741	15	1500	1443	49067652	0.0117	4763	13	1502	1444
$FLG10_3$	40675587	0.0091	2258	20	1506	1460	36050416	0.0113	6733	12	1503	1471
$FLG10_8$	46257990	0.0088	6371	15	1501	1433	44331128	0.0110	5424	16	1500	1422
$FivePoints_3$	536853	0.0272	6801	1	111	98	470481	0.0527	5394	1	155	154
$FivePoints_5$	393334	0.0262	5577	1	109	108	376620	0.0796	5568	2	220	216
$PhyllisLeeper_3$	5589066	0.0309	6306	1	72	59	5571034	0.0845	4831	1	187	171
$PhyllisLeeper_5$	4949207	0.0278	2131	1	107	100	4840641	0.0439	4092	1	136	135
WLC_3	4635768	0.0151	4753	1	116	108	4629039	0.0174	1528	1	122	119
WLC_7	4180389	0.0147	4927	1	121	119	4146478	0.0174	4758	1	190	184
$BearTown_3$	4368877	0.0452	4540	1	116	100	4337020	0.0965	4523	1	169	162
$BearTown_5$	4510859	0.0355	2907	1	103	101	4249328	0.0561	5095	1	129	101
$Kittaning4_3$	1675397	0.0536	745	1	55	50	801334	0.1940	124	2	123	116
$Kittaning4_5$	13388788	0.0536	24	1	55	53	936671	0.2567	2982	1	147	142

Table 6: Net present and index values, CPU time, number, total area and total core area of habitats in the last period for the keep-efficient solutions with the best net present value or the best connectivity index.

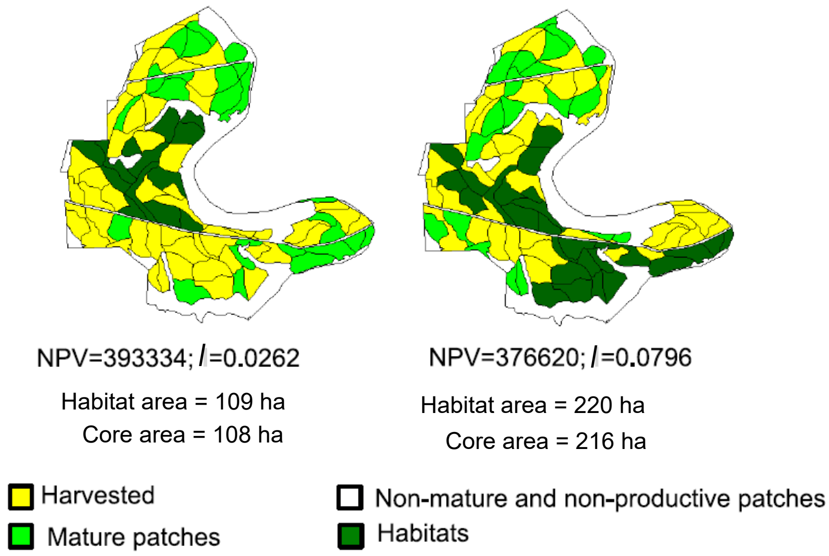

Figure 9: Map representing the solutions obtained for FivePoints$_5$ with the best net present value or the best connectivity index, concerning the last period of the planning horizon.

7 Conclusions

This work deals with crucial spatial issues that affect forest fragmentation caused by harvesting: total habitat area, total core area inside habitats and connectivity between habitats. Habitat area and core area are important to provide life needs for specific plant communities and wildlife species. Connectivity is considered a key issue for the conservation of biodiversity and maintenance of natural ecosystem stability and integrity. We consider a bi-objective harvest scheduling problem that maximizes both the net present value and the inter-habitat connectivity. Spatial constraints of the problem are on the maximum clearcut area and on the habitats' total area and total core area. Core area is modeled directly using the concept of subregions, and inter-habitat connectivity is measured by means of the probability of connectivity index. Larger values of the connectivity index are expected to enhance the connectivity between habitats (by reducing the inter-habitat distances), as well as the area of all habitats (by increasing the number of habitats or the area of each habitat).

We developed a multi-objective Monte Carlo tree search (MO-MCTS) to find a set of efficient solutions of the problem. MCTS is used as an alternative to standard binary tree search, in situations where the construction and storage of the whole tree is computationally expensive, mainly for medium and large instances. This approach was tested with forests ranging from 32 to 1363 stands, and temporal horizons ranging from three to eight periods. We ran MO-MCTS 12 times for each instance, each time allowing MO-MCTS to run for two hours at the most.

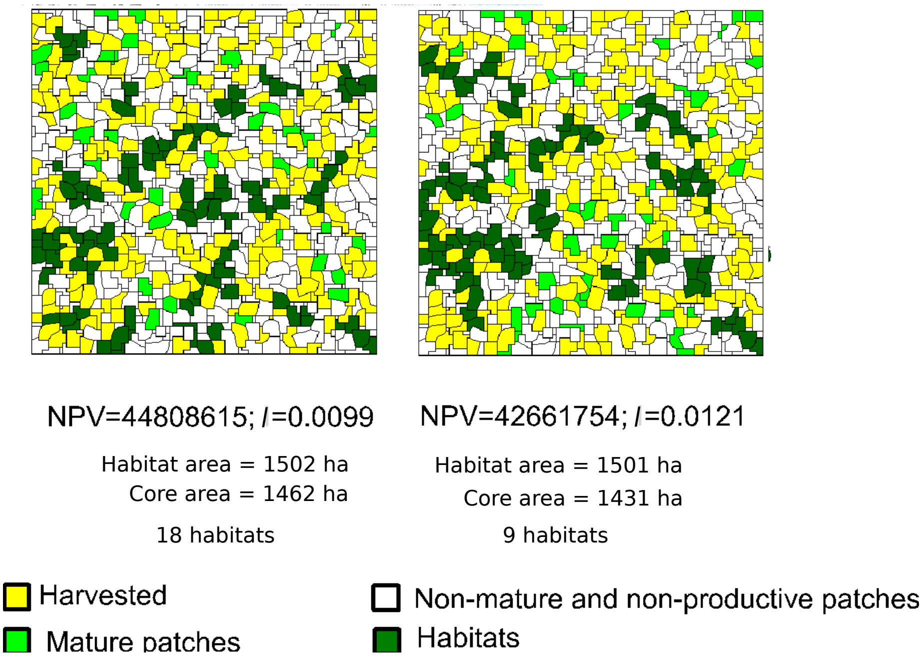

Figure 10: Stand map representing the solutions obtained for FLG9_3 with the best net present value or the best connectivity index, concerning the last period of the planning horizon.

The structure of MO-MCTS allows us to know the sequence of non-dominated points that are being generated. Due to the stochastic nature of the method, each run usually gives different set of efficient solutions. The average number of efficient solutions obtained for each instance ranges from 2.50 to 13.58. This average decreases significantly when we select the efficient solutions obtained in all runs (keep-efficient solutions). This means that running MO-MCTS several times allows us to discard false-efficient solutions, which may provide better approximations to the optimal Pareto front.

With respect to the trade-off between net present value of timber harvested and probability of connectivity index, we observed that ensuring a better inter-habitat connectivity is not always obtained at the expense of a reduction in the net present value. By comparing the best solutions in terms of connectivity index with the best solutions in terms of net present value (among the keep-efficient solutions), we found that the total area of habitats increased for almost all instances, which is not surprising, and the total core area of habitats increased with the total area of habitats. The number of habitats decreased for the majority of the large instances, while for the small and medium instances, this number increased or did not change.

MO-MCTS can be used to assist decision makers by providing several efficient alternative solutions, thus allowing him/her to choose the one with the best tradeoff between net present value and inter-habitat connectivity, according to his/her own preferences. Further work to improve the ability of the algorithm to generate more efficient solutions and to solve much larger instances is needed.

A great merit of MO-MCTS is that its structure can be easily adapted to include other issues, so further work may also include other forest management concerns.

A Fixing parameter values

How some parameters and data of the model 2-18 were fixed:

Parameter α, the maximum deviation of the harvested volume in each period from the target volume, was set to 0.15.

Parameter v_0, the target volume for each period, was calculated as the timber volume of all stands in the first period divided by the total number of periods, that is, $\mathsf{v}_0 = \sum_{i=1}^{n} \mathsf{v}_{i1}/T$ (see, *e.g.*, [Yoshimoto and Brodie, 1994]). In order to obtain a feasible solution, Neto et al. [2013] replaced the denominator of v_0 by $T+1$ for Bear Town$_5$, WLC$_3$ and PhyllisLeeper$_5$, $T+2$ for Bear Town$_3$, PhyllisLeeper$_3$, FLG9$_8$ and FLG10$_8$, Stafford and El Dorado, and $T+3$ for FLG9$_3$ and FLG10$_3$.

The minimum average age (measured in periods) for the forest at the end of the planning horizon $\mathsf{Age}_{\mathsf{end}}^{\mathsf{min}}$ was set to $(T+1) - T/2$.

In the case of WLC, the coefficients of the objective function (2), the net present values npv_{ct}, included the residual value of the stands to perpetuity. For all instances, the annual discount rate used to calculate the net present values was 4%.

In order to obtain a feasible solution or to make the constraints of total core area or total habitat area active, different values for the minimum total habitat area in each period ($\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}$) and for the minimum total core area in each period ($\mathsf{C}_{\mathsf{tot}}^{\mathsf{min}}$) were experimented, giving the values displayed in Table 2.

With respect to the probability of connectivity index I_t, for the calculations of g_{hr}, we assumed that wildlife has, in average, a dispersal probability of 0.5 for 5 Km.

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Chapter 6

Conclusions

The main focus of this thesis was to develop mathematical models and methods in integer programming for solving harvest scheduling problems with environmental restrictions. The objective of these restrictions is to reduce the impact of harvesting activities on wildlife, soil, water, forest aesthetics, among others.

A common approach for incorporating environmental restrictions into harvest scheduling problems is to include spatial constraints, addressing the relative arrangement of stands and the interconnections among them. This approach adds substantial complexity to the problems and solution techniques.

Four different type of constraints were taken into account for this purpose, not all at the same time, leading to the definition of three different harvest scheduling problems. These constraints state (1) a maximum on the clearcut area and a minimum on the (2) total habitat area, (3) total core area inside habitats and (4) inter-habitat connectivity.

Constraints on maximum clearcut area prevent large continuous harvested areas. However, this type of constraints may promote a dispersion of small clearcuts across the forest, contributing to a fragmented forest. Fragmentation refers to a reduction in habitat area and inter-habitat connectivity. Habitats may become too small for many species, in terms of total area or core area, to meet the living needs of these species. The distances between habitats may be greater than species could travel, introducing barriers to species movement.

Constraints imposing a minimum in the total habitat area contribute to increase the number or the area of habitats. Nevertheless, large habitats may contain a small amount of core area if they are elongated or irregularly shaped. Core area constraints tend to limit the impact of fragmentation on the amount of core area inside the habitats. Connectivity constraints contribute to shorten the inter-habitat distances.

As the direct use of general purpose solvers is not possible to solve the forestry problems of this thesis for large instances, branch-and-bound and Monte Carlo tree search were the methods proposed and designed specifically for these problems. Branch-and-bound and Monte Carlo tree search could explore potentially all the solution space on the integer variables, and hence provide an optimal solution, or work as heuristics, if some criterion, as time limit or solution quality, is achieved. The methods worked as heuristics, with a time limit of two hours, for the majority of the instances.

Both methods use a binary tree search of sequential decisions, where each decision corresponds to harvesting or not a stand in a given period. Each solution at a child node is evaluated by taking the solution at the parent node and considering the changes in the stand in which the decision is taken, providing a considerable speedup of the methods. Monte Carlo tree search is used as an alternative to the standard binary tree search, where the construction and storage of the tree is computationally expansive, mainly for medium and large instances.

The three models developed for the forestry problems are based on the so-called cluster formulation, one of the three basic formulations described in the literature for the harvest scheduling problems with maximum clearcut size constraints. Cluster formulation yields better linear programming bounds for these problems than the other two formulations, the so-called path and bucket formulations, which is crucial for the effectiveness of branch-and-bound.

Constraints on the minimum total habitat area for each period are considered in all models, although based on different definitions of habitat. While in Paper 1, a habitat is a mature patch with a minimum area requirement, in Papers 2 and 3, a habitat is a mature patch with a minimum core area requirement.

The inter-habitat connectivity is modeled using a non-linear measure, the so-called probability of connectivity index. This index satisfies a set of properties that any ideal connectivity index should fulfill. Moreover, it is proved in Appendix C that the index assumes non-increasing values in the binary tree search, another important property for the implemented branch-and-bound. Models in Papers 1 and 3 address inter-habitat connectivity 1) with constraints that impose a minimum value for the index in each period (Paper 1) and 2) with an objective that maximizes the minimum value of the index over all periods (Paper 3).

Core area is directly measured using the concept of subregions. The subregions are created by buffering each stand outward, the stand's impact zone, and intersecting the impact zones with the stands. Each subregion is core area if there are no harvest activities in the stand where the subregion is and in the stands that influence the subregion with their impact zones. Models in Papers 2 and 3 address core area with constraints that impose a minimum value for the total core area inside habitats in each period.

Models in Papers 1 and 2 aim at maximizing the net present value. The first model is non-linear, due to the probability of connectivity index, and the second is linear but it requires more variables, those related with the subregions. Model in Paper 3 is a bi-objective non-linear formulation with the same variables as the second model. The objectives are the maximization of the net present value and the maximization of the inter-habitat connectivity.

Branch-and-bound is used to solve the first and second models and Monte Carlo tree search is applied to the third model, giving the multiobjective Monte Carlo tree search.

The proposed models and methods were tested with sixteen real and hypothetical instances ranging from small to large (forests from 32 to 1363 stands and temporal horizons from three to nine periods).

The results obtained for branch-and-bound and multiobjective Monte Carlo tree search show that these methods were able to find solutions for all instances. For almost all instances, branch-and-bound was able to find good solutions (within 1% of the optimum) or the optimum, in reasonable time, for the first and second problems. Multiobjective Monte Carlo tree search could only identify a small subset of efficient solutions. The average number of efficient solutions ranged from 2.5 to 13.58. This number decreased significantly when the efficient solutions obtained in all runs were selected (keep-efficient solutions). This means that the multiobjective Monte Carlo tree search needs to be run several times, to provide better approximations to the optimal Pareto front.

With respect to the forestry fragmentation caused by harvestings, the results suggest that, although clearcut size constraints tend to disperse clearcuts across the forest, compromising the development of large habitats, close to each other, the proposed models, with the other environmental constraints, attempt to mitigate this effect.

All models can help to increase the total habitat area. The first and third models can help to increase the connectivity between habitats. It is proven (Paper 3) that if the connectivity index requirement for a given period is not less than a specific value (the square of the ratio of the total habitat area requirement to the total area of the forest), than the total habitat area constraint is satisfied in that period. Although connectivity and total habitat area constraints promote the occurrence of larger habitats, core area constraints are needed to provide larger core areas inside habitats. The second and third models can help to increase these core areas. The third model is also useful to assist the decision maker in estimating efficient alternative solutions and the corresponding trade-offs between the net present value and inter-habitat connectivity.

As would be expected, the restrictions on inter-habitat connectivity, total habitat area and total core area reduced the net present value generated by the harvestings. However, the results from the different papers suggest that it is possible to address the environmental restrictions with small reductions.

6.1 Future directions

Further research to improve the ability of the branch-and-bound and Monte Carlo tree search algorithms to solve much larger instances is needed. It seems reasonable that the number of efficient solutions obtained by Monte Carlo tree search could also be increased.

Designing forest management approaches for harvest scheduling problems with timber and environmental concerns is a major challenge in forest planning. Further work may also include other environmental requirements. The structure of branch-and-bound and Monte Carlo tree search methods may be easily adapted to include these concerns. Additionally, Monte Carlo tree search may be easily adapted to include other environmental objectives.

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Appendix A

Forest policies and laws

This appendix focuses on some forest policy and legislation addressing environmental concerns of a subset of seven world's countries. The selection of the countries was made on the basis of the importance of the forest sector in each country or in the world, but also considering the availability of data and information. The countries are: Australia [Mayers and Bass, 1999, DAWR2013, 2013, FPCT, 2015, DAWR2017, 2017]; Canada [NBCF, 2005, NRC2009, 2009, NRC2017, 2017]; Finland [Lier and Parviainen, 2013, METSO, 2015, Simonsson, 2016]; Russian Federation [FCRF, 2006, WWF, 2017]; Sweden [Mayers and Bass, 1999, SFI, 2009, Timonen, 2011, SFISFC, 2012, Simonsson, 2016]; Portugal [Mayers and Bass, 1999, Branco et al., 2014, Feliciano et al., 2015, DGRF, 2015, ICNF, 2017]; United States of America [USDA2015, 2015, USDA2017].

UNCED introduced a statement of principles for a global consensus on the management, conservation, and sustainable development off all types of forests. It have led to a substantial expansion of international law that influences increasingly national policy and law development, focus mainly on the protection and sustainable forest management. A forest policy provides a goal or a direction about forests and their use, usually not specifying in detail the instruments or practices to implement it. Forest legislation is one key instrument for implementing the policy. National forest programmes denote a forest policy framework built on a number of specific principles where the policy can be reflected or not in the forest legislation. According to Food and Agriculture Organization of the United Nations (FAO), more than 140 countries announced about development or approval of their national forest policies, 76 countries approved or revised their national forest policies after 2000 [FAO, 2015, WWF, 2017]. The national forest policies adopted by many countries differ considerably from one country to another. Even if there are considerable differences between the various forest policies and laws, most important is the similarity in the main goals for the protection and sustainable management of forests and principles for achieving those objectives.

In addition to the forest policies and laws, the forest sector pursues voluntary forest certification. Forest management certification was emerged in the 1990s as a voluntary tool to promote sustainable forest management and trade of products coming from sustainably managed forests. By using forest certification, the producers can demonstrate a commitment to the environment that the wood is coming from sustainable forestry. Certification is usually verified from an independent third party inspector (the certifier) that gives a written assurance that the quality of forest management practiced by a defined producer conforms to specific standards. Forest certification is usually in addition to national forest policies. The two major international certification schemes are the Forest Stewardship Council (FSC) and the Program for the Endorsement of Forest Certification (PEFC). Forest certification offers different benefits to different groups: consumers can consider certification in their buying decisions; forest companies can use certification to show they are responsible resource managers; public can look to the value of certification in improving forest practices around the world. As forest certification has developed as a way of demonstrating the implementation of sustainable forest management practices, most governments also encourage forest owners/managers to use forest management certification. Thus, forest management certification complements forest management laws and regulations of the countries.

Table A.1 shows the forest area as a percentage of country and global land area for the selected countries in 2015 [FAO, 2015]. It also compares the percentage of forest area designated for conservation of biodiversity in 2015 [FAO, 2015] and the percentage of forest certified area in 2010 [FAO, 2015]. The forests of Russian Federation, Canada, United States of America and Australia are immensely important for the present and future of humanity as they account about 40% of the worlds forest cover. Concerning to the percentage of forest area designated for biodiversity conservation, Australia and United States of America rank highly among the countries studied. Finland, Sweden and Canada lead the world in percentage of area certified. Finland and Sweden have more than 50% of forests certified. Canada does not have a large percentage area designated for biodiversity conservation, but a vast area of forest is certified. Russian Federation have the smallest percentages of area designated for biodiversity conservation and forest certified area.

Country	Forest area (1000 ha)	Forest area within country land area (%)	Forest area within global forest area (%)	Forest area designated for biodiversity conservation (%)	Forest area certified (%)
Russian Federation	814931	49.8	20.3	3.3	3.3
Canada	347069	38.2	8.7	6.9	44.1
United States of America	310095	33.8	7.8	20.9	7.6
Australia	124751	16.2	3.1	21.2	8.7
Sweden	28073	68.4	0.7	16.3	68.9
Finland	22218	73.1	0.6	15.1	93.6
Portugal	3182	35.3	0.08	5.9	19.7

Table A.1: Forest area as a percentage of country land area and global forest area. Forest area designated for conservation of biodiversity and forest certified area as a percentage of country forest area.

For each country, an overview of forest ownership (private and public) and forest practices system is presented. When an ownership type is highly majority, this analysis is restricted to that forest ownership. Since forest governance is largely handled at sub-national level in Canada, United States of America (US) and Australia, the analysis was assessed in five Canadian provinces (Quebec, Ontario, British Columbia, Alberta, New Brunswick), five US states (Louisiana, Washington, Oregon, California, Alaska) and five Australian states (Queensland, New South Wales, Western Australia, Victoria, Tasmania). Taking into account the data and information available, fourth criteria have been selected for a more detailed analysis:

- Maximum clearcut sizes - existence or not of mandatory limits on the size of clearcuts. These requirements limit the contiguous area that can be harvested in a particular period or in a sequence of periods.
- *Allowable cut (AC)* - requirements on the volume of timber that may, or must, be harvested annually (annual allowed cut) or periodically (over a five- or ten-year period) from a specified area. It can be achieved with either an evenor an uneven flow harvest. In forests intended to sustain wood production over the long term, it cannot exceed the *sustained yield* of wood the forest is capable of producing. The sustained yield is the equilibrium level of production from the growth rate of trees comprising a forest, annually or periodically, in perpetuity. It means the continuous production with the aim of achieving a balance between net growth of a forest and harvest. A non-declining even flow of timber reduces harvest volumes in the short term to those that can be steadily sustained over the long term. In all case studies, annual allowed cut (AAC) approaches are based on a sustained yield, a non-declining even flow and a balance of economic, social and environmental factors.
- Reforestation - requirements of reforestation following harvesting, including the specification of minimum stocking levels (growing space occupancy of trees) and minimum time frames to achieve these targets. Reforestation may be accomplished either naturally or by planting. The retention of individuals or groups of high-value trees in the cutting areas (referred as *green-tree retention*) enhances the feasibility of natural regeneration, which may be considered desirable for the preservation of local genetic variation. Nevertheless, planting reduces the risk of regeneration delay and permits more control over stocking density, species composition, and genetics, so planting is frequently done with the green-tree retention.
- Protected areas - strategies to maintain or increase the effectiveness and size of protected areas. Connecting protected areas, such as national parks and wilderness areas, as well as other crucial habitats, ensures larger, cohesive landscapes of high biological integrity that allow for the migration, movement, and dispersal of wildlife and plants [Ament et al., 2014]. The importance of increasing connectivity in fragmented forest landscapes by using protected areas is a topic of discussion today [Timonen, 2011].

Canada has the third largest forest area in the world, with a vast boreal forest. Forests play a vital role in Canada's economy, since it is one of the world's largest producers and exporters of wood products. The majority of Canada's forest land, about 94%, is publicly owned and managed by provincial, territorial and federal governments [NRC2017, 2017]. Although only 6% forests are privately owned, they are highly productive. The publicly owned lands (provincial, territorial and federal) are commonly referred to as *Crown lands*. As a result of the high majority of forest lands are publicly owned, this analysis is restricted to this forest ownership. Private large forests are notably in the provinces of Quebec, Ontario, British Columbia and New Brunswick [McDermott et al., 2010, NRC2017, 2017]. Sustainable forest management of publicly owned lands is a rigorous process, supported by laws, regulations and policies. Forest Acts (forest legislative documents) have been enacted in all provinces back to the 19th century. They are implemented to set aside protected areas, protect wildlife, specify harvesting and regeneration practices, prevent illegal logging in Canada and the import of illegal timber products into Canada.

Clearcutting in publicly owned lands is usually made by private companies operate under a licence or timber supply agreement which has to be approved by local authorities. These forest licenses or agreements require a forest management plan, usually covering a period of several decades (*e.g.* 80 years in New Brunswick), that align with strategic regional land use plans overseen by the province. A number of jurisdictions along the provinces of Canada established limits for the size of the clearcuts according to regional characteristics. On provincial lands, British Columbia imposes a maximum clearcut size of 40 ha for the coastal and southern interior regions, and 60 ha for northern interior regions. Alberta's limits are 24 ha for spruce areas and 100 ha for pine and deciduous areas. Ontario province requirements are much less restrictive, with maximum clearcuts of 260 ha. In Quebec, the clearcut policy is particularly complex, prescribing clearcut limits, but also requiring that clearcuts be of small size across a set percentage of the harvested area. The maximum limit is 150 ha [McDermott et al., 2010]. Hardwood and softwood clearcuts in New Brunswick do not exceed 100 ha in size. The time for harvesting in adjacent clearcuts (referred as *green-up* time in the literature) must not be less than two periods (one period is equal to five years) when the combined area of adjacent clearcuts exceeds 100 ha. For the purpose of maintaining the AAC, clearcut sizes may be range between 80 to 125 ha and the green-up time must not be less than one period [NBCF, 2005].

Approaches to AAC regulations vary according to the Canadian province. In Ontario, Quebec and Alberta, AACs are set for management units (areas designed to manage timber). In Ontario and British Columbia, AACs are calculated based on a range of economic, social and environmental factors. Quebec uses a simulation model which incorporates a large variety of factors, including sustained yield and non-timber objectives. AACs are calculated based on perpetual sustained yield in Alberta. In New Brunswick, the stated goal is to maximize the sustained yield rather than ensure even flow of timber [McDermott et al., 2010].

All Canadian provinces studied have mandatory reforestation requirements, with specification of minimum stocking levels and times frames to achieve these targets [McDermott et al., 2010].

Protected areas are a major component of Canada's national forest conservation strategy. In these legally defined areas, some activities are restricted in order to preserve natural ecosystems. Harvestings, mining and hydroelectric development are banned in nearly 95% of protected forests [NRC2017, 2017]. All provinces studied have developed strategies to increase the size and effectiveness of their protected areas. These strategies have focused greatest attention on the protection of representative habitats. For example, in New Brunswick, six old-habitats were defined, based on the requirements of the vertebrate species assigned to them. Minimum patch sizes and inter-patch distances were defined in terms of these requirements. Minimum requirements on habitats sizes range from 10 ha to 375 ha. Any harvesting in that areas must maintain the old forest condition (tree species composition, structure and minimum patch size) [NBCF, 2005]. There are also been coordination across jurisdictions with British Columbia and Alberta to develop inter-provincial protected areas. At the national level, the Canadian Council on Ecological Areas provides all information about all protected areas. In addition, Canadian civil society activists have been an important role in the creation of new protected areas. One illustrative example is the mountainous area of British Columbia (about 6.4 million ha) dubbed as *Great Bear Rainforest* [McDermott et al., 2010]. Quebec's new strategic guidelines in 2011 highlight the importance of consolidating its network of protected areas by maintaining or improving connectivity between the different protected areas [NRC2017, 2017].

The **United States of America** has the fourth largest forest area in the world. US is one of the major forest products producer and it is the world's largest consumer of wood products. A slight majority of the US forest land is private owned. The US Department of Agriculture (USDA) Forest Service is responsible for governing the majority of federal forests (referred to as national forests). Forest practices are governed by a wide variety of state laws, regulations and policies ranging both across land ownership types and states. Four state studied, respectively, Washington, Oregon, California and Alaska have enacted some form of general Forest Act. In Louisiana, this analysis is restricted to private owned lands, the vast majority ownership. In general, Forest Acts require, at a minimum, notification of timber harvest, a submission of management plans or harvest permits. The national forests are managed by a succession of Forest Acts, regulations and federal environmental legislation. The 1976 National Management Forest Act limits the size of clearcuts and sets the requirements for land management plans in the national forest system. Each national forest must prepare a forest-wide management plan involving extensive public input and including timber inventory, sustained yield levels, strategies for the protection or rehabilitation of endangered species and habitats, and strategies for the provision of multiple goods and services.

The forest harvesting systems for public and private forests vary by state. Clearcut size limits on private lands in California range between 8.1 and 12.1 ha depending on harvest methods, with permission in some cases for 16 ha. The requirements in Washington and Oregon are 48.5 ha, with sizes up to 97 ha possible with approval. In Alaska and Louisiana there are no limit requirements [McDermott et al., 2010]. Limits on national forests vary according to geographic areas, forest types, or other suitable classifications. Limits are 24.3 ha for Douglas fir forest type of California, Oregon, and Washington, 40.5 ha for the hemlock-Sitka spruce forest type of coastal Alaska, and 16.2 ha for all other forest types. Limits larger than those specified may be permitted in some situations, as a result of natural catastrophic condition such as fire, insect and disease attack, or windstorm [USDA2017].

The National Forest Management Act 1976 sets a process for determining the *annual sale* *quantity* (US term for AAC) based on requirements for sustaining multiple forest uses, environmental protection and non-declining even flow over a 15-year planning period. On private lands, Louisiana, Washington, Oregon and Alaska do not have policies. California include provisions concerning the maximum sustained production of high quality timber products [McDermott et al., 2010].

Reforestation is a mandatory requirement in Washington, Oregon, California, Alaska private lands and national forests. Louisiana imposes harvest plans for timber harvest only on commercial forest lands, with requirements for reforestation [McDermott et al., 2010].

US has a long story of protected areas development leading to a large protected area. Alaska ranks in the top in terms of land area under protected status. The 1973 US Endangered Species Act provides a program for protecting endangered species and their critical habitats. US was a pioneer in the creation of national parks, with dual objectives of protection and recreation. The US Geological Survey Gap Analysis Program, launched in 1989, had increasingly served to coordinate federal, state and private efforts in biodiversity conservation. In 2009 was published the first database of federal and state conservation lands, where the lands are classified by codes according to the level of human disturbance allowed. The 2012 Planning Rule includes requirements for managing for ecological connectivity on national forest lands [USDA2017]. Civil society initiatives have been played a key role in driving forest conservation, including some partnerships between US states, and US and Canada. While there are many catalysts for the establishment of protected areas, the retention of federal lands is the largest predictor of protected areas. With regard to private lands, there has been a considerable growth in the uses of lands to protection, due to the inclusion of specific biodiversity conservation measures and the quality of auditing [McDermott et al., 2010].

Australia has about 98% of native forests, being Eucalypt the dominant forest type [DAWR2013, 2013]. Forestry activity in Australia occurs substantially on public land (case study). About 39.6% of native forest is under lease-hold and 27.2% is privately managed. So, about 66.8% of native forests is under some form of private management [DAWR2017, 2017]. Since 1930s policy encouraged the pulp and paper industry and since the 1970s promoted woodchip exports. This led to widespread clear-felling and regeneration of even-aged crops, with loss of biodiversity. The beginning of a revolution in Australian forest policy was introduced in 1992 by the National Forest Policy Statement. It provides the framework within which the governments (Australian government and states governments) work cooperatively to achieve their vision for sustainable management of Australia's forests, while ensuring that community expectations are met [Mayers and Bass, 1999]. To achieve their vision for the forest eleven broad national goals must be pursued. Almost all Australian states have codes of forest practices applied to private and public lands. In addition to the National Forest Policy Statement, Australia has a number of key forest policies to achieve key conservation and management outcomes for Australia's forest and forest industries, such as: *Plantations for Australia: the 2020 Vision*, *National Indigenous Forestry Strategy*, *Illegal logging*. As forest certification has developed as a way of demonstrating the implementation of sustainable forest management practices, Australian Government also encourages forest owners/managers to use forest management certification.

With respect to clearcut sizes limits (usually referred as *coupe*), Tasmania imposes as maximum of 100 ha on slopes not greater than 20°, that is twice that allowed on steeper slopes in that state (slope $> 20°$). A maximum of 40 ha for mature karri forests and 20 ha for regrowth karri forests is imposed in Western Australia. Victoria's limits are 40 ha per year for wet eucalyptus forests and 120 ha over 5 years for the same type of forests. Queensland and New South Wales Sales do not have requirements on public lands [McDermott et al., 2010].

The volume that could be harvested from public native forests was regulated in all Australian states studied. A maximum AAC is specified for a given region or species group over a period, typically 5-15 years. The AAC is based on the predicted sustained yield with a non-declining even-flow of various categories of wood products in the region, estimated over a long-term period (*e.g.* 90 years in Tasmania) and making allowance for various risk and uncertain factors, such as fire [McDermott et al., 2010].

Reforestation is required in all case study. The most common method is natural reforestation. Minimum stocking levels are specified as also the period within which regeneration must be achieved in almost all states. The exceptions are Queensland' native forests where no specific time limit is expressed. For example, in Western Australian karri forests, a stocking level of 1666 species per hectare is required on 75% of areas within 18 months of harvesting and on 100% of areas within 30 months. In Tasmania and Victoria different stocking standard are specified for different types and silvicultural systems [McDermott et al., 2010]. Regeneration should occur over one or two years following harvesting in Tasmania.

Current forest management practices in Australia provide a balance between timber production requirements and the protection of biodiversity, which has become a fundamental objective of native forest management. One of the eleven broad national goals is the conservation of an extensive and permanent native forest estate to conserve the full suite of economic, social and environmental values for current and future generations. A key element of the approach adopted in the National Forest Policy Statement is the Regional Forest Agreement (RFA) between the Australian Government and a state government, a 20-year plan for the productive use and conservation of Australia's native forests. Ten RFAs were progressively signed between 1997 and 2001. Under each RFA, the Australian Government has accredited the Australian's states to deliver sustainable forest management. In 2013, the Australian Government committed to maintaining its support for long-term RFAs by seeking to extend and establish 20-year rolling lives for each RFA. Some practices in Tasmania to maintain habitat diversity are the retention of wildlife habitats strips and patches containing trees with nesting hollows and other old growth structure elements in areas to harvest [FPCT, 2015].

Russian Federation has by far the largest forest area of any country of the world, including the world's greatest expanse of intact boreal forest. Grand part of Russian forests are in remote and in inaccessible zones (*e.g.* in Siberia and Russian Federation Far East), being largely undisturbed and having high biodiversity and wilderness values. Forest sector is of global importance of timber production, carbon cycle, and biodiversity. Almost all forests are publicly owned and grow on land of the forest state, reason why this analysis is restricted to this forest ownership. Until 2006, the forest legislative document was the Forest Code of the Russian Federation, which was approved in 1997. In 2006, a new Forest Code was adapted. Sustainable forest management, biological diversity conservation in forests, and enhancement of their potential is referred to as a key principle of the Forest Code (Article 1). The basic territorial administration/management units are forest districts (*lesnichestvo*) and municipal forest parks (*lesopark*). According to the new Forest Code, each province must be developed a forest plan, covering all forest districts, municipal forest parks and specify zones for use and conservation. The process of definition and adoption of a formal Russian forest policy document is ongoing since 2011. In September 2013 the new policy document called *Foundations of the State Policy in the Field of Forests Use, Conservation, Protection and Regeneration till 2030* has been adopted. This document defines principles, goals, and tasks related to forests use, conservation, protection and regeneration. It serves as a basis for further formulation of regulations, plans and other policies.

According to the harvesting rules, wood may be harvested by individuals and legal entities, under lease agreements or sale/purchase contracts. Clearcut size limits range according the forest group type or region: for protection forests (group 1), 5-10 ha for coniferous and broad-leaved and 15 ha for pioneer hardwoods; for populated area forests (group 2), 10-20 ha for coniferous and broad-leaved and 25 ha for pioneer hardwoods; for remote and production forests (group 3), 25 ha for pine and 25-50 ha for other types; 250 ha for pioneer hardwoods in Far East. For mountain forests, the limits for the different forest groups must be 1.5 times smaller on 11-20° slopes and 2 times smaller on 21-30° slopes than in corresponding plain forests. [McDermott et al., 2010]. Dead, damaged (stands damaged with fire, wind, snow, pests and other adverse factors) and over-mature stands shall be the first to be made available for wood harvesting. It is prohibited to harvest wood earlier than at the ages of cutting, leave trees designated for cutting, cut and damage trees to be preserved, destroy or damage poles demarcating the boundaries, compartments, cutting areas.

AAC is regulated and calculated for each forest district and national forest park based on a non-declining even flow. A significant part of the standing forest included in the allowable cut is economically inaccessible due its remoteness from transportation routes or small growing stock. Furthermore, large volumes of timber remain on the ground in harvest areas. This leads to volumes of timber harvested in Russia well below the AAC [McDermott et al., 2010].

Reforestation with stocking levels and times frames are regulated. Stocking densities of at least 4000 seedlings are required on dry soils and densities of at least 6000 seedlings per hectare are required in the forest-steppe zone. If the regeneration is carried out by planting seeds, the planting density must be increased by 20%. If regeneration is accomplished using transplants, the planting density is 2500 seedlings per hectare, providing that final prescribed stocking levels are reached. The time to reach reforestation and the required stocking levels vary according to forest group type and harvested method. For group 1 and 2, forests with clearcutting, stocking should be reached within 1 or 2 years. For group 1, the time frame is 2 to 5 years in adjacent cuts, and 1 to 4 years in group 2. For group 3, conifers must be regenerated within 1 to 3 years in adjacent cuts and hardwoods within 1 year [McDermott et al., 2010]. Harvesting prescriptions requiring green-tree retention in the cutting areas for the purposes of regeneration [FCRF, 2006].

Forests within specially protected nature areas are the forests within state nature reserves, national parks, nature parks, nature monuments, state special-purpose reserves and other specially protected nature areas established by federal laws. In specially protected forests, harvesting is limited or prohibited. The new Forest Code contain provisions for the protection of habitats of rare and endangered wildlife species. Within cutting areas, it is not permitted to cut viable trees of high-value species, occurring at the outskirts of their natural ranges [FCRF, 2006]. Biodiversity preservation is also considered in the 1995 On Wildlife Act and 2002 On Environmental Protection Act. The On Wildlife Act includes habitat protection and requires that any economic activity that impacts wildlife habitat must include mitigating measures [McDermott et al., 2010]. An important step in the development of planning mechanisms for biodiversity conservation was made with adoption of the Russian Federation's Governmental Programme ”Environment” for 20122020, including the subprogramme ”Biodiversity in Russia”. Priorities of this subprogramme are defined as development and effective functioning of the protected areas network and conservation and restoration of rare and endangered species of animals and plants.

Sweden is a country dominated by forests. Forestry is crucial for the national economy and most Swedes closely relate to forests and forestry pursuits. More than 80% of Sweden's forests are privately owned (case study), 57% by small-scale private owners (usually family enterprises) and 24% by corporations, and 19% state-owned [McDermott et al., 2010]. Small-scale private forest owners have drawn their livelihoods from both agriculture and forestry on their properties. The first forestry Act in Sweden was established in 1903. The forest laws up to the 1950s had considered the well being of the small-scale private forest owners, and between 1960 and 1990 had served the interests of the industrial owners [Mayers and Bass, 1999]. Clauses related to retention forestry have been included in the Swedish Forestry Act since 1975 [Timonen, 2011]. A major change was advocated in 1993 by the Swedish Forestry Act in which the production and the environment objectives are placed on a par in forest policy. Clearcutting in Sweden is commonly referred to as regeneration felling. The key message of the 1993 and subsequent Acts was that felling must be followed by regeneration [Mayers and Bass, 1999]. The Forestry Act was complemented in 1999 by the Environmental Code with encompasses biodiversity conservation and environmental protection [McDermott et al., 2010]. The Swedish Parliament, in 2010, adopted 16 national environmental quality objectives to achieve by 2020, several concern forestry as a rich diversity of plant and animal life, thriving wetlands, natural acidification only, reducing climate impact and sustainable forests, a magnificent mountain landscape [SFI, 2009, SFISFC, 2012].

Forest management plans are voluntary. Clearcuts are limited to 20 ha in alpine areas. State permission is required for clearcuts exceeding 0.5 ha. If permission is obtained, there are no upper limits for the size of regeneration feelings in lowland areas [McDermott et al., 2010].

There are no requirements for the calculation of AAC [McDermott et al., 2010].

Regeneration is a mandatory requirement after felling with regulations on stocking levels and time frames. The Swedish Forestry Act requires that regeneration must have been completed by the end of the third year after felling. The Act also sets on stands minimum rotation ages between regeneration fellings (number of years required to establish and grow timber for regeneration felling), from 45 to 100 years for coniferous forests. For forest areas larger than 50 ha, half of the area is allowed to be clearcuts and stands younger than 20 years [McDermott et al., 2010].

Beside the general conservation considerations in forest management, the designation of more strictly protected forests areas is another strategy in the Swedish model to protect forest biodiversity. Forests can be formally protected as national parks, nature reserves, habitat protection areas or under nature conservation agreements. The government decides national park status, whilst nature reserves are designated by county and local administration. The Swedish Forest Agency designates habitat protection areas and nature conservation agreements. The most protected forest land is located in national parks and nature reserves. In protected areas, timber extraction is not allowed unless it is to specifically improve the value of the land for nature or for the purposes of cultural conservation. In addition, all unproductive forestland is protected under the Swedish Forestry Act. The term woodland key habitat was launched in Sweden in 1990. The main idea behind this concept is to conserve biodiversity in production forests by delineating and preserving small habitats that are supposed to be particularly valuable for maintaining landscape-level biodiversity [Timonen, 2011, Simonsson, 2016]. Key habitats do not have any formal protection today, but instead have a strong informal protection through certification. The Swedish Forest Agency manages a database of all key habitats in the country. They have an average size of 3.4 ha on private land and 8.0 ha on public land [Simonsson, 2016]. As they are generally relatively small, the sizes of species populations are necessarily rather limited within a key habitat. The small size may also result in isolation and strong edge effects. Species with poor dispersal abilities are also not likely to benefit greatly from key habitats in the connectivity perspective. Consequently, the small size of habitat keys has been the main cause of criticism towards the use of habitats keys as a conservation tool. The identification of core areas is a key approach for the County Administration's work on green infrastructure [Timonen, 2011, Simonsson, 2016]. Since the middle of the 1990s, forest companies also have been an important role in voluntary forest protection. It is estimated that the large industrial forest enterprises now leave some 10% of the potential harvest standing for ecological reasons [SFI, 2009]. In Sweden, all Natura 2000 areas are protected with the support of the Environmental Code.

Finland has the highest percentage of forest land within the country land area of the countries studied and of all countries in the Europe. About 60% of Finland's forests are owned by non-industrial private landowners, 26% by the state, 9% by forest industries and 5% by others (municipalities, parishes and other public corporations). Small-scale non-industrial private ownership is predominant. Finnish forestry is commonly denominated by family forestry, as forest management is undertaken by families in own forests [McDermott et al., 2010]. Relative to its size, Finland is more dependent on forest sector than any other country in the world. As a consequence, Finland has accumulated an expertise in forestry and industrial manufacturing of forest products that is unique in Europe. Most of the products of forest industries are exported, being the European Union the most important market [Lier and Parviainen, 2013]. The principal laws on forestry are the 1996 Forest Act and the associated 1996 Act on the Financing of Sustainable Forestry. The Forest Act regulates forest practices and the associated Act provides public grants or loans to private landowners to promote forest conservation and sustainable management. Policies are implemented by a network of 13 regional forestry centres. Metsähallitus is a state enterprise operating within the Ministry of Agriculture and Forestry, responsible for sustainable and profitable forestry activities on state lands. Private forest planning is undertaken by the regional centres.

Management plans are voluntary for private lands. No size limits for clearcuts are specified for private and public forests, although the most private owners precludes large clearcuts [McDermott et al., 2010].

The annual cut volumes are calculated at national level, regional centres, forest owners but there are no policies for the calculation of AACs [McDermott et al., 2010].

Reforestation is a mandatory requirement on all ownership types, with regulations on stocking levels and time frames. The stocking levels depending on species, habitat type and region [McDermott et al., 2010].

Owing to many protection programs and decisions, the area of protected forests has tripled in Finland over the past decades. The establishment of statutory conservation areas has been based on conservation programmes for national parks, strict nature reserves, mires, waterfowl habitats, herb-rich forests, shorelines and old-growth forests adopted by the governments [Lier and Parviainen, 2013]. Biological diversity in forests is promoted by means of forest legislation, recommendations and instructions for best practices in forest management. The Forest Biodiversity Programme for Southern Finland 2008-2025 (METSO) aims at activating voluntary-based conservation agreements between forest owners and public authorities through a payment for ecosystem services mechanism. Forest owners get full financial compensation equivalent to the value of timber at the protected site. With permanent protection, the private forest owner's income from the site is tax free. Additionally, protected sites can be used for nature based tourism and recreation. The Finnish government's objective by 2025 is to have sites covering about 96000 ha that will be voluntarily offered by landowners to be established as private nature reserves or that will be acquired by the State. At the end of 2014, about 52% of the first target had been achieved, including 37000 ha offered by landowners and 13000 ha protected by the State [METSO, 2015]. As a result of the programme, a total of 1.300 new protected areas were established in private forests in 2005-2011 [Lier and Parviainen, 2013]. The Finish Environment Institute and the 13 centres are responsible for promoting conservation at regional level. Metsähallitus is the entity responsible for managing state-owned nature reserves and other state protected areas [McDermott et al., 2010]. The Nature Conservation Act lists nine protected habitat types, three of which are found in forests. The Forest Act contains definitions of key habitats [Lier and Parviainen, 2013]. The average size of habitat keys is 0.7 ha [Simonsson, 2016]. There is legislation for the protection of species and conservation of habitats classified under the EU Habitat Directive [McDermott et al., 2010].

Forest is the dominant land use in **Portugal**, occupying 35% of the territory. This places Portugal within the average of the EU countries. Plantation production is most significant in Portugal. About 85% of forest and woodland is under private ownership (case study), where small-scale is predominant (93% of these forests are less than $10ha$). Pulp and paper companies own 6%, local communities 7% and the state 2% [McDermott et al., 2010]. The Portuguese forestry sector plays an important role in the national economy, representing around 10% of exports and 2% of Gross Value Added (GVA), which is only surpassed in EU by Finland and Sweden [DGRF, 2015]. The Portuguese Forest Service was established in 1886 and the first national laws were promulgated in 1901, 1903 and 1905, applied similarly to private and public forests. The main priorities are the protection and reforestation. The 1938 Forest Law was significant in enabling subsequent forestation by the state. Afforestation and reforestation through the 1970s, 80s and 90s supported by public incentives lagged far behind the area of deforestation due to forest fires and have not taken up most of the land released from agriculture due to farm outmigration. European Community and the World Bank have subsidized afforestation with eucalyptus. Forest fires are the most direct cause of deforestation in Portugal. A great effort is put into forest fire fighting every year, and prevention has been emphasized and underlined by institutions that struggle to decrease the effects of this cause on deforestation in Portugal. The access to the fire sites and to water resources is the major difficulty when fighting forest fires [Mayers and Bass, 1999, Branco et al., 2014]. The relevant policy documents in the Portuguese constitution states that "the state will promote forestry policies according to ecologic and social circumstances" (93rd article, point two) [Feliciano et al., 2015]. A forestry policy was adopted in 1996, which led to the elaboration of the Sustainable Development Plan for the Portuguese Forest, in 1998. The National Forest Strategy approved in 2006 constitutes a reference element of the guide lines of public and private action plans for the development of the forest sector. National Forest Strategy was updated in 2015 assuming as main goal the sustainability of forest management. The guide lines are: minimization of the risk of fire and biotic agents; territory specialization (goals as the protection of forest areas of high natural value, other is to support the installation of forest areas); production improvement through forest sustainable management systems; internationalization and increase the products' value; overall improvement of the sector's efficiency and competitiveness; rationalization and simplification of the political instrument [DGRF, 2015]. The National Forest Strategy recognizes the importance of forest certification for sustainable forest management. One of the main barriers to certification is the high costs for small-scale forestry.

There are no legal limits for clearcut sizes. Legal restrictions are on the harvesting of immature pine and eucalyptus (Law-Decree no.$173/88$). In harvest areas not less than 2 ha of maritime pines, 75% of the trees must be more than or equal to 17 cm in diameter at base height or a perimeter higher than $53cm$. For eucalyptus trees in harvest areas not less than 1 ha, 75% of the trees must be more than or equal to 12 cm in diameter at base height or a perimeter higher than 37.5 cm [McDermott et al., 2010].

Although there are no reforestation requirements, this practice is encouraged [McDermott et al., 2010].

There are no requirements for the calculation of AAC. However, best harvest practices in Portugal suggest the planning of harvest volumes considering a number of ecological constraints. A guideline is that for long rotation cycles (30-40 years), selective logging should be conducted every four to five years for cleaning and ensuring the maximum production of the stand. For even longer cycles, selective cuts should be mad even seven to eight years [McDermott et al., 2010].

Law-Decree no.19/93 outlines seven different classes of protected areas. Classifications include national park (*e.g.* Park Peneda-Gerês), natural park (*e.g.* Serra da Estrela) and natural reserve (*e.g.* Serra da Malcata). The main establishment and management of protected areas (of national interest) is a competency of the Institute for Nature Conservation and Forests (ICNF). Protected areas of regional/local interest are established under proposal of, and managed by local authorities (municipalities), with the participation of ICNF. Law-Decree no.142/08 also allows the creation of private protected areas, based on the application of respective property owners. The *Faia Brava* reserve was legally recognized in 2010 as the first national private protected area within a specific legal framework. National and private protected areas are included in the National network of protected areas of responsibility of ICNF. The designation of the areas for the network Natura 2000 is also a responsibility of ICNF. The conservation of habitats classified under the EU Habitat Directive is regulated by Law-Decree no.93/90. There is legislation for the protection of cork and holm trees (Law-Decree no.169/01 amended by Law-Decree no.155/04). Portugal is involved in several bilateral cooperation initiatives involving Spain to reinforce biodiversity management efforts (*e.g.* Iberian-lynx recovery in Iberia and establishment of the transboundary Park Gerês-Xurés, involving the National Park Peneda-Gerês in Portugal and the National Park Baixa Limia-Serra do Xurés in Spain) [ICNF, 2017].

Table A.2 summarizes the forest policies and laws for maximum clearcut sizes, AAC requirements and reforestation criteria. The following sources for the areas of provinces/states were consulted: NRC2009 [2009] for forest and other wood land area of Canadian provinces; USDA2015 [2015] and DAWR2013 [2013] for forest area of US sates and Australian states, respectively. Portugal is the country with less policies concerning the criteria selected. Relatively to the mandatory requirements for maximum clearcut sizes, limits range from 5-10 ha in Russia Federation to 260 ha in Ontario. Maximum clearcut sizes are in general relatively small in Russia Federation relative to those of Canada, another country with large boreal forest. Annual allowable cuts are required by the largest number of jurisdictions, which are calculated based on sustained yield, a non-declining even flow and a variety of economic, social and environmental factors. Reforestation policies that require the stocking levels or time frames are found in almost all case study.

	Forest	Maximum clearcut sizes (ha)		AAC		Reforestation	
	area (10^6 ha)	private	public	private	public	private	public
Quebec	84.60		150		sustained yield		stocking levels time frames
Ontario	68.29		260		various factors		stocking levels time frames
British Columbia	64.25		60 (northern) 40 (coastal, southern)		various factors		stocking levels time frames
Alberta	36.39		100 (pine, deciduous) 24 (spruce)		sustained yield		stocking levels time frames
New Brunswick	6.21		100		sustained yield		stocking levels time frames
Alaska	0.052	no limits	40.5 (hS spruce) 16.2 (other types)	no requir.	even flow	time frames	stocking levels time frames
California	0.013	8.1 - 12.1	24.3 (Douglas fir) 16.2 (other types)	sustained yield	even flow	stocking levels time frames	stocking levels time frames
Oregon	0.012	48.5	24.3 (Douglas fir) 16.2 (other types)	no requir.	even flow	stocking levels time frames	stocking levels time frames
Washington	0.009	48.5	24.3 (Douglas fir) 16.2 (other types)	no requir.	even flow	stocking levels time frames	stocking levels time frames
Louisiana	0.006	no limits		no requir.		commercial forests	
Queensland	51.03		no requir.		even flow		stocking levels
New South Wales	22.68		no requir.		even flow		stocking levels time frames
Western Australia	19.22		40 (m karri) 20 (r karri)		even flow		stocking levels time frames
Victoria	8.19		120 (we 5years) 40 (we year)		even flow		stocking levels time frames
Tasmania	3.71		100 (slope$\leq 20°$) 50 (slope$>20°$)		even flow		stocking levels time frames
Russia Federation	814.93		250 (FE pHdw) 25-50 (g3 other) 25 (g3 pine) 25 (g2 pHdw) 10-20 (g2 cbl) 15 (g1 pHdw) 5-10 (g1 cbl)		even flow		stocking levels time frames
Sweden	28.07	20 (alpine)		no requir.		stocking levels time frames	
Finland	22.22	no limits	no limits	no requir.	no requir.	stocking levels time frames	stocking levels time frames
Portugal	3.18	no limits		no requir.		no requir.	

Table A.2: Forest area, clearcut sizes limits, AAC (annual allowable cut) and reforestation requirements; hS spruce - hemlock-Sitka spruce; g1, g2 and g3 - forest group 1, 2 and 3, respectively; FE - Far East; pHdw - pioneer hardwoods; cbl - coniferous and broad-leaved; m karri - mature karri; r karri - regrowth karri; we 5years - wet eucalypt per 5 years; we year - wet eucalypt per year; no requir. - without mandatory requirements.

Appendix B

Methods

This appendix gives an overview of the methodology used in this thesis. It begins with basic notions in graph theory and integer programming. Then an introduction to tree search methods for combinatorial optimization is presented, followed by a description of branch-and-bound and Monte Carlo tree search.

B.1 Graphs

A graph is one of the fundamental data structures used in computer science. A wide range of algorithms is available to compute properties and measures in graphs. The definitions and results described in this section are standard and may be found in any introductory text on graph theory; see, for example, Diestel [2012].

A *graph* $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a nonempty set of *vertices* or *nodes* $\mathcal{V}$ and a set of *edges* $\mathcal{E}$, each of which is a pair of vertices of $\mathcal{V}$. If the pair is ordered, it is called an *arc* or *directed edge*; otherwise the pair is called an *undirected edge*, or simply an *edge*.

A graph in which every edge is ordered is called a *directed graph* or *digraph*. If every edge is unordered, the graph is called an *undirected graph*, or simply a *graph*. Graphically, vertices are usually represented by points. In an undirected graph, an edge $\{v', v\}$ is represented by a line. In a directed graph, an arc (v', v) is represented by an arrow from u to v, where v' and v are called the initial and terminal vertices, respectively.

A *simple graph* is an undirected graph without multiple edges or loops (*i.e.*, there is at most one edge between two vertices, and there are no edges joining a vertex to itself). The following definitions concern simple graphs.

Two vertices are *adjacent* when there exists an edge between them. A graph is said to be *complete* if every vertex is adjacent to all other vertices. A complete graph of n vertices is denoted by K_n.

A *bipartite graph* is a graph whose vertices can be partitioned into two disjoint subsets in such a way that every edge connects vertices from one subset to the other. A *complete bipartite graph* is the bipartite graph where each vertex from one subset is adjacent to all vertices in the other subset. A *complete bipartite graph* where each subset has n vertices is denoted by $K_{n,n}$.

A *subgraph* $\mathcal{H} = (\mathcal{V}', \mathcal{E}')$ of graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a graph such that $\mathcal{V}' \subseteq \mathcal{V}$ and $\mathcal{E}' \subseteq \mathcal{E}$. The *subgraph induced by* $\mathcal{V}'$ is a subgraph whose vertices are given by $\mathcal{V}'$ and whose edges consist of all edges in $\mathcal{E}$ between those vertices.

A *clique* is a complete subgraph. A clique is *maximal* when it is not contained in another clique in the graph.

A graph is *planar* if it can be drawn in a plane without its edges crossing.

Theorem B.1.1 (Kuratowski) *A graph is planar if and only if does not contain any subdivision of K_5 or $K_{3,3}$.*

As a result of Kuratowski's theorem, a graph is planar if and only if there are no cliques with more than four vertices.

A *path* in a graph is a sequence of edges such that there is a vertex in common among every two successive edges. A path between v_1 and v_n can be represented by

$$\{\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}\}$$

or simply by $\{v_1, v_2, v_3, \ldots, v_{n-1}, v_n\}$.

A path that begins and ends in the same vertex is called a *cycle*.

A graph is *connected* if for every pair of distinct vertices there is a path between them. A *connected component* of a graph is a maximum connected subgraph (*i.e.*, which is not contained in any other connected subgraph).

A *weighted graph* is a graph with weights associated to its edges. An unweighted graph implicitly has a weight of one on each edge. The *distance* of a path is the sum of the weights of its edges.

B.1.1 Trees

A *free tree*, or simply a *tree*, is a connected simple graph with no cycles. A *rooted tree* is a tree in which a node is designated the *root*. In a rooted tree, the *depth* or *level* of a node v is the distance of the path from the root to v. The root has depth 0. The *height* of a rooted tree is the maximum depth of its nodes.

If a node v immediately precedes v'' on the path from the root to v'', then v is the *parent* of v'', and v'' is the *child* of v. A *leaf* is a node with no children.

An m_b-*ary tree* $(m_b \geq 2)$ is a rooted tree in which every node has m_b or fewer children. A *complete* m_b-*ary tree* is an m_b-ary tree in which every node either is a leaf or has exactly m_b children, and all leaves have the same depth. An *ordered tree* is a rooted tree in which the children of each node are assigned a fixed ordering. A *binary tree* is an ordered 2-ary tree in which each child node is designated as *left-child* or as *right-child*.

A *balanced tree* is a tree where the difference between the depth of any two leaves is no more than a certain value.

Theorem B.1.2 *The complete binary tree of height* hg *has* $2^{hg+1} - 1$ *nodes.*

Corollary B.1.3 *Every binary tree of height* hg *has at most* $2^{hg+1} - 1$ *nodes.*

B.2 Integer programming

This section summarizes basic concepts and results in linear and integer programming. For a thorough introduction to integer programming see, *e.g.*, Wolsey [1998].

Linear programming (LP) deals with solving optimization problems characterized by a linear objective function and a set of linear constraints. Integer programming (IP) and mixed integer programming (MIP) differ from LP since they require, respectively, all or some variables to be integers. In binary integer programming (BIP), the integer variables are restricted to the values 0 or 1.

Let $\max_{x \in X} \ f(x)$ be an LP problem, where $f(x) = \mathsf{c}x$ and $X = \{x : Ax \leq \mathsf{b}, x \in \mathbb{R}^n\}$, with $\mathsf{c} \in \mathbb{R}^n$, $A \in \mathbb{R}^{m' \times n}$ and $\mathsf{b} \in \mathbb{R}^{m'}$. The set X of feasible solutions is called the feasible region or solution space.

An IP problem is a LP problem with the inclusion of integrality constraints. Hence, the IP problem can be stated as $\max_{x \in X} \ f(x)$, where $X = \{x : Ax \leq \mathsf{b}, x \in \mathbb{Z}^n\}$. The LP problem obtained by ignoring the integrality of constraints is usually called the LP relaxation. The solution space of the LP relaxation contains the solution space of IP; hence, the optimum of the relaxation is an upper bound on the optimum of the corresponding IP. If the LP relaxation is infeasible, then IP is also infeasible. A point $x \in \mathbb{R}^n$ is a *convex combination* of p' points $x_1, x_2, \ldots, x_{p'}$ points in $\mathbb{R}^n$ if $x = \sum_{i=1}^{p'} \alpha_i x_i$, with $\alpha_1, \ldots, \alpha_{p'} \geq 0$ and $\sum_{i=1}^{p'} \alpha_i = 1$. If additionally all $\alpha_i > 0$, the convex combination is called *strict*.

A set $S \subseteq \mathbb{R}^n$ is a *convex* set if, given any two points $x_1, x_2 \in S$, any convex combination of x_1 and x_2 is also in S.

The *convex envelope* or *convex hull* of $S \subseteq \mathbb{R}^n$, denoted by $\mathrm{conv}(S)$, is the set of all convex combinations of the points of S.

A set $S = \{x \in \mathbb{R}^n : Ax \leq \mathsf{b}\}$ described by a finite set of linear constraints is a *polyhedron*.

An *extreme point* of a polyhedron S is a point which does not belong to a strict convex combination of any two other points in S.

Theorem B.2.1 *Every polyhedron is a convex set.*

Proposition B.2.2 *The convex hull of the feasible solution space of an LP problem is a polyhedron.*

Theorem B.2.3 *For an LP problem with extreme points in the feasible region, if the optimum is finite, then there is an optimal solution which is an extreme point.*

Corollary B.2.4 *If two or more different points are optimal, then any point which is a convex combination of these points is also optimal.*

The previous results play a key role in the solution of LP problems. The fact that the search of an optimum can usually be limited to the set of extreme points is exploited by one of the best known methods for solving LP problems, the simplex method.

IP problems are usually much more difficult to solve than LP problems, because the set of feasible solutions is no longer convex. Typical methods to solve IP problems, such as branch-and-bound and cutting-plane techniques, involve enumeration.

The quality of a formulation of an IP problem with feasible space X can be judged by the closeness of the feasible set of its LP relaxation to $\mathrm{conv}(X)$. Let A and B be two formulations of the same IP problem and let X_A and X_B be the feasible sets of the corresponding LP relaxations. Formulation A is considered better than B when $X_A \subset X_B$.

B.3 Tree search

Tree search is a technique that systematically explores the solution space of a problem, with the aim of finding an optimal solution. This technique enumerates solutions on a tree structure, ensuring that an optimal solution (or a set of efficient solutions, in multi-objective optimization) will be found. It is particularly suitable for problems that may be represented as trees of sequential decisions. The technique is mostly effective when it is possible to find bounds for the solutions that can be obtained under a node, which allow eliminating nodes of the tree without loss of optimality. The process of deleting the nodes and all of its potential children from the tree is called *bounding* (which may be further categorized into *fathoming* and *pruning*). This is the most important component of a tree search method, since it prevents the tree from growing too much.

Tree search methods can be used as exact or heuristic methods. In the latter case, search is interrupted on the base of a time limit, an iteration limit or a limit size for the tree. Consequently, the solution space is not fully explored, and solutions found may not be optimal (or not efficient, in the case of multi-objective optimization).

B.3.1 Branch-and-bound

Branch-and-bound (B&B) is a widely used tree search method for solving difficult combinatorial optimization problems. An early version of the core B&B method was provided by Lawler and Wood [1966]; a recent survey is available in Morrison et al. [2016].

B&B enumerates all relevant solutions by storing partial solutions on the tree structure. Each node of the tree corresponds to a subproblem obtained from the original problem, and children are generated by partitioning the solution space into smaller regions through the addition of restrictions to the subproblems, while ensuring that an optimal solution will be found.

B&B is a very general framework, which has to be filled out for each specific problem type. Even though there are many variants, the overall structure of branch-and-bound has three main components: *node selection*, a *bounding function*, and *branching* (or *partitioning*).

Let $\max_{x \in X} f(x)$ be an optimization problem P, where X is the solution space and f is the objective function. Let $X' \subseteq X$ be a solution subspace relative to a subproblem P'. The tree initially consists of a single node, the root, which is stored in a queue data structure Q. For many problems, a feasible solution can be produced in advance using a heuristic method; otherwise, search starts with an empty solution, and a starting feasible solution is searched throughout the execution of the method. At a given moment, the current best-found solution, x^*, is denominated the *incumbent*.

At each iteration of a B&B algorithm, a node is selected for exploration from Q using some selection strategy (node selection component), explained latter. Let k be the node selected and X^k the corresponding solution subspace. The next step in an iteration is usually the calculation of a bound for the subproblem (bounding function component), *i.e.*, a bound for the solutions that can be found in X^k; let B^k be this bound. If it can be established that there are no feasible solutions in X^k, or if B^k is not better than the incumbent, *i.e.*, if $B^k \leq f(x^*)$ (in maximization problems), then this node can be neglected; we say it is *fathomed*. Otherwise, the node's solution space is divided into two or more subspaces (branching component). The branching procedure partitions the feasible region X' into m_b subregions X'_1, $X'_2, \ldots, X'_{m_b}$. The m_b new nodes are stored in the queue Q. A pruning step may discard nodes in Q that will never be neither feasible nor optimal. If node k is a leaf and corresponds to a feasible solution $x \in X'$ of the original problem, its objective value is compared to the objective of the incumbent. If $f(x) > f(x^*)$, the incumbent solution is updated.

If it is used as an exact method, the B&B procedure is repeated until the queue Q is empty. The optimal solution of the problem is taken as the final incumbent solution. The main steps of B&B are presented in algorithm 1.

The time complexity of B&B algorithms is related to two factors: the *branching factor* of the tree m_b, which is the maximum number of children generated at any node in the tree, and the height of the tree hg. Thus, any B&B algorithm operates in $\mathcal{O}(Mm_b^{hg})$ worst-case running time, where M is a bound on the length of time needed to explore a subproblem

[Morrison et al., 2016]. However, pruning rules may substantially improve the algorithm's performance.

Algorithm 1: Main steps of branch-and-bound

Step 1 **Initialization:** create a queue Q with the root node
set $f^* := -\infty$
Step 2 **Termination:**
if $Q = \emptyset$ **then**
  if $f^* = -\infty$ **then** the problem is infeasible
  else solution x^* is optimal
Step 3 **Node selection:**
if $Q \neq \emptyset$ **then**
  select and remove a node k from Q
Step 4 **Bounding function:**
calculate bound B^k
Step 5 **Pruning:**
if $B^k \leq f^*$ **then** go to Step 2
if *node is a leaf* **then**
  if *node's solution x is feasible and $f(x) > f^*$* **then**
    update $x^* := x$
    update $f^* := f(x)$
  go to Step 2
Step 6 **Partitioning:**
create branches $k' = 1, \ldots, m_b$
for $k' = 1, \ldots, m_b$ **do**
  insert branch k' in Q
go to Step 2

The three main components are now explained. An application of B&B to integer programming is presented next.

B.3.1.1 Node selection

The node selection strategy determines the order in which unexplored subproblems are explored, or in other words, it guides the search on the tree. The choice of the strategy has potentially significant consequences for the amount of computation time required, as well as the amount of memory used. Common search strategies are *depth-first search*, *breadth-first search* and *best-first search*; see, *e.g.*, Morrison et al. [2016].

In depth-first search (DFS), from a given point in the tree, a child node is successively chosen to be processed, going deeper in the enumeration tree, until a leaf is reached; when this happens, the search backtracks. The most recently generated subproblem is the next to be explored. The search is thus implemented through a last-in-first-out (LIFO) process on the queue. DFS is often preferred when time is limited, since it typically reaches a feasible incumbent solution early on. The number of subproblems that need to be stored is bounded by the number of levels of the search tree, hence requiring low memory. Another advantage is that in DFS we can often reuse information from the parent node, again with savings in the memory used. A problem which may arise with DFS, particularly on unbalanced trees, concerns wrong choices taken early in the search; it may take a prohibitively long time until these choices are reconsidered.

Breadth-first search (BrFS) explores all nodes at a given level before processing any nodes at the next level. BrFS is the opposite of DFS in that it is implemented with a first-in-first-out (FIFO) process on the queue. BrFS has the advantage of always finding an optimal solution that is closest to the root of the tree, thus operating well on unbalanced search trees. The drawback is that, as the number of nodes at each level of the search tree grows exponentially with the level, BrFS requires a large amount of memory, especially for larger problems. *Beam search* (BS) overcomes this limitation; it is a particular case of BrFS, where the number of solutions to search per level is limited by a given constant. This strategy reduces both time and memory requirements relatively to BrFS, but does not guarantee finding optimal solutions.

Another kind of node selection strategy is possible when there is heuristic information for evaluating the quality of the nodes in the queue. The most common method in this case is best-first search (BFS), where typically the node with the best value of the bound is chosen. An advantage of BFS is that it is able to find good solutions earlier in the search process; this has a direct impact on the size of the search tree. BFS may be implemented by means of a queue ordered according to the bound of each of its nodes; when the bound for the best node in the queue is worse than the objective value of the incumbent, search can be stopped.

B.3.1.2 Bounding function

A bounding function is commonly used to determine if a node can be discarded. The most common approach is determining an upper-bound (for a maximization problem) on the objective function value of each subproblem, leaving out some constraints of the subproblem (relaxation). For example, some integrality constraints are relaxed in each subproblem in integer programming (see Section B.3.1.4). Other examples are the relaxation of constraints which are difficult to formulate mathematically, or of constraints which are too expensive to compute.

The bounding function is a key component of any B&B algorithm, in the sense that a low quality bounding function cannot be compensated for by means of good choices of branching and selection strategies. The goal is to provide a value for the bound close to the optimum of the subproblem, while using only a limited amount of computational effort. One often experiences a trade-off between quality and time when dealing with bounding functions. More time spent on calculating the bound may lead to a better bound value, which usually results in a smaller search tree.

B.3.1.3 Partitioning

Partitioning consists in generating children of a given node, associated to candidate subproblems, typically by limiting the domain of variables used in branching (*branching variables*). Therefore, as the search moves down in the tree, the feasible region of the generated descendants becomes more restricted.

The choice of a branching rule determines how children are generated from a given node. Binary branching rules focus on subdividing each subproblem into two smaller subproblems, while non-binary strategies create more than two subproblems. An example of a binary rule is presented next.

B.3.1.4 Example: Application to integer programming

One common application of the B&B procedure is on the solution of integer programming (IP) problems, where all the variables must be integer. If integrality constraints are relaxed, we obtain the linear programming (LP) relaxation. When the relaxation at the root node is integer, it is the optimum of the IP problem. Otherwise, fractional variables are successively removed by limiting their domain, thus creating two LP subproblems. For each subproblem, the optimum of the LP relaxation is the classic bounding function.

Branching decisions are imposed by adding constraints to each of the subproblems, shrinking their feasible region, but keeping all the integer feasible solutions in the tree. The standard branching rule selects a fractional variable x_i in solution of the LP relaxation of a subproblem, and creates two new branches by adding the restrictions $x_i \leq \lfloor \xi \rfloor$ and $x_i \geq \lfloor \xi \rfloor + 1$, where ξ is the value of x_i and $\lfloor \xi \rfloor$ represents the greatest integer less than ξ.

If all variables have integer values, a feasible integer solution is obtained. The first integer solution found becomes the incumbent; later integer solutions may replace it if their objective is superior. An optimum of the IP problem is the incumbent when the search terminates.

In the particular case of binary variables, the two branches are generated by considering $x_i = 0$ in one child, and in the other $x_i = 1$. Variables not fixed previously are allowed to take on any value of the range between zero and one.

B.3.2 Monte Carlo tree search

Monte Carlo tree search (MCTS) is a recent tree search technique that builds a tree in memory, making use of the outcomes of stochastic simulations. Browne et al. [2012] provides a survey of MCTS methods.

MCTS is composed of a number of iterations, each of which is split up into four main steps: *node selection*, *expansion*, *simulation* and *backpropagation*. In an iteration nodes are selected starting from the root (or initial state) and the focus recursively descends (node selection step) to navigate to a node in the tree which is not fully expanded (a leaf node). Once at a leaf node, MCTS creates one or more new nodes (expansion step) and for each child node it applies a simulation (simulation step), where variables are successively fixed until a solution is reached (terminal state). Each iteration ends updating of statistics through the visited nodes upwards, until reaching the root node (backpropagation step). This process is repeated until a termination criteria is met, such as reaching a given number of iterations or elapsed time. A great benefit of MCTS is that, during a simulation, the values of intermediate states do not have to be evaluated; only the value of the terminal state at the end of each simulation is required. These simulations are used over a large number of iterations to selectively grow a tree. Since simulations do not take long to perform, MCTS is able to quickly explore search spaces. This is what gives MCTS the advantage over deterministic methods in large search spaces. The steps of MCTS are depicted in Algorithm 2. Each of the main steps is now described more in detail.

Algorithm 2: Main steps of Monte Carlo tree search

```
Step 1 Initialization:
       set f* := −∞
       create root node s
Step 2 Termination:
       if computational budget is depleted then
           if f* = −∞ then nothing can be said
           else propose heuristic solution x*
Step 3 Node selection: starting from node s
       repeat
           recursively select child k with maximum U(k)
       until k is a leaf in the current tree
Step 4 Expansion: let C be the set of children obtained from expanding k
       for each element k' ∈ C do
Step 5     Simulation: let z be the result of a simulation from k'
Step 6     Backpropagation: propagate z up the tree until reaching s
           if z > f* then
               update f* := z
               update x* := solution at the end of this simulation
       go to Step 2
```

B.3.2.1 Node selection

The selection starts from the root node and iteratively chooses the child node which currently looks more promising to descend through the tree until reaching a leaf node. The definition of promising is one of the key aspects determining the performance of MCTS. In game-playing, average win rate is used for node selection. The Upper Confidence Bounds for Trees (UCT) algorithm [Kocsis and Szepesvári, 2006] provides an enhancement to this simple rule, by considering the selection of a child node as a *multi-armed bandit* problem, from game playing.

Bandit problems are sequential decision problems, in which selections are made amongst K actions in order to maximize the cumulative reward by consistently taking the optimal action. The term bandit refers to the usual name of a Casino's slot machine (one-armed bandit). In a multi-armed bandit problem there is a finite number of independent slot machines, where each machine has a fixed unknown expected return. The player iteratively selects a machine (pulls an arm). A K-armed bandit problem is defined by random variables $X_{k,s'}$, $1 \leq k \leq K$ and $s' \geq 1$, where each $X_{k,s'}$ denotes the reward that is incurred when arm k is pulled at play s'. For arm k, the rewards $X_{k,s'}$ are considered independent and identically distributed, with unknown mean and variance. For simplicity, variable $X_{k,s'}$ is usually assumed to lie in the interval $[0, 1]$.

The multi-armed bandit problem may be approached using a policy that determines which bandit to play based on past rewards. Auer et al. [2002] proposed a simple policy called UCB1. This policy dictates to play arm k that maximizes:

$$U(k) = \overline{X_k} + \sqrt{\frac{2\ ln\ s'}{s_k}}, \tag{B.1}$$

where $\overline{X_k}$ is the average reward from arm k, s' is the overall number of plays so far and s_k is the number of times arm k was played. The first term encourages the exploitation of higher-reward choices, while the right term encourages the exploration of less visited choices.

Considering the choice of a child node, when descending a MCTS tree, as a multi-armed bandit problem, the value of each child of a node is the expected reward approximated by Monte Carlo simulations; hence these rewards correspond to random variables with unknown distributions. Kocsis and Szepesvári [2006] proposed the use a node selection policy called UCB1. This policy dictates for each child k:

$$U(k) = Q(k) + \sigma\sqrt{\frac{ln\ s_{p(k)}}{s_k}}, \tag{B.2}$$

where σ is a constant that balances both terms in the sum, $s_{p(k)}$ is the number of simulations done under the parent node $p(k)$, and s_k is the number of simulations done under the child node k (*i.e.*, simulations started from k or any node in the subtree under k). In game-playing, $Q(k)$ is typically taken to be the average reward of simulations run from k. Equation (B.2) balances the exploitation of areas which appear to be promising (first term) and the exploration of a new portion of the search space (second term). The value of constant σ can be adjusted to lower or increase the amount of exploration performed, but it is common to find $\sigma = \sqrt{2}$. At each node, the child with maximum $U(k)$ is selected, until an unexpanded node is reached.

Applying MCTS to solve optimization problems has two significant differences to its application in game-playing. One difference concerns to the evaluation of nodes and their associated statistics. Whereas in game-playing a branch with a high average win rate is suggestive of a strong line of play, in optimization, the average rate under a node is not a good estimator of the optimal solution to the node's underlying subproblem. Another difference is that in game-playing the rewards are in the range $[0, 1]$ (0 for losing, 1 for winning) and objective functions may take arbitrary values. Since (B.2) was designed with rewards in the $[0, 1]$ interval, in order to maintain the proper balance between the two components of (B.2), Pedroso and Rei [2015] propose the following shape for function $Q(k)$:

$$Q(k) = \frac{e^b - 1}{e - 1}, \quad b = \frac{\widehat{w}^* - \overline{z}_k}{\widehat{w}^* - \widehat{z}^*}, \tag{B.3}$$

where $\widehat{z}^*$ and $\widehat{w}^*$ are, respectively, the best and the worst simulation results found in the part of the tree explored so far, and $\overline{z}_k$ is the average outcome of simulations under node k.

B.3.2.2 Expansion

The expansion step adds nodes to the leaf of the MCTS tree selected as detailed above. Because for most domains the whole search tree cannot be stored in memory, an expansion strategy decides, for a given node k, whether this node will be expanded by storing one or more of its children in memory. Two strategies for node expansion could be considered [Pedroso and Rei, 2015]:

Single expansion – a single child node is created using a randomly chosen unexplored decision in k. Other unexplored decisions are kept for a later time when node k is again selected for expansion.

Full expansion – all children of node k are immediately created by generating all possible decisions in the node.

B.3.2.3 Simulation

From each node created in the expansion step, a simulation is performed, until a solution is reached. Various approaches can be taken. The simplest approach consists in taking uniform random decisions that require nothing more than a generative model of the problem. Heuristic construction algorithms incorporate domain-specific knowledge, and typically allow for faster convergence at the expense of simplicity and generality.

A simulation strategy is subject to two trade-offs. One trade-off is between the search and knowledge. Adding knowledge to the simulation, the simulations become more accurate and their results more reliable. However, if the heuristic knowledge is too computationally expensive, the number of simulations per second may decrease too much and, consequently, the MCTS search tree will be shallow. Another trade-off deals with exploration versus exploitation. If the strategy is too stochastic, too much exploration takes place, causing that the simulations to be unrealistic. In contrast, if the strategy is too deterministic, too much exploitation may take place. The exploration of the search space becomes too selective, causing that the simulations to be biased, and thus not representative of the potential of the subtree under the node in question.

B.3.2.4 Backpropagation

Backpropagation is the step that communicates the outcome of the simulation backwards, from the selected node until the root. This step updates statistics on all nodes in the path between the selected node and the root by increasing the visit counts and modifying the simulation scores. Backpropagation ensures that the values of each node accurately reflect simulations performed in the subtrees that it represents; this information will be used to inform future tree policy decisions.

Appendix C

Probability of connectivity index

This appendix is about the index that was used to quantify the inter-habitat connectivity. The properties of the index described in Saura and Hortal [2007] are summarized. A property concerning the monotony of the index within the search trees of the branch-and-bound and Monte Carlo tree search methods is deduced.

Several connectivity indices, substantially different in their measurement, have been applied to measure landscape connectivity. However, despite their widespread use, there is still a lack of comprehensive understanding of their sensitivity to pattern structure and different spatial changes, which limit their interpretation and use. A connectivity index should be sensitive to all changes that can occur in the landscape. Furthermore, among the different changes that can occur, the index would be able to discriminate which of those changes are more relevant.

The choice of the index called *probability* of connectivity was motivated by its good performance relatively to a set of properties that any connectivity index should fulfill. Saura and Pascual-Hortal compared this index with other connectivity indices through a set of desirable properties and it was the only that satisfied all properties. According to these properties, the index: 1) has a predefined and bounded range of variation (between 0 and 1); 2) computes both on vector and raster data; 3) is insensitive to subpixel resampling of landscape pattern; 4) indicates lower connectivity when the distance between habitats increases; 5) attains its maximum value when a single habitat covers the whole landscape; 6) indicates lower connectivity as a habitat is progressively more fragmented; 7) considers negative the loss of a habitat or part of a habitat; 8) detects as more important the loss of bigger habitats; 9) is able to detect the importance of the stepping-stones; 10) is able to detect as less critical the loss of stepping-stones that leave most of the remaining habitat area still connected; 11) is unaffected by the presence of adjacent habitats, *i.e.*, the overall value of the index is not affected by the partition of a habitat into several habitats. The first three properties are desirable for landscape metrics in general and the remaining properties refer to the prediction of the index to detect spatial changes that may affect the landscape and to detect the most relevant landscape elements.

In addition to all these properties, the probability of connectivity index could be easily adaptable to a wide range of situations with different levels of detail and data availability in the connectivity analysis, both for characterizing inter-patch connections (trough Euclidean distances, minimum cost distances, etc.) and mature patches attributes (patch area, habitat quality, core area, etc.).

The inter-habitat connectivity was addressed in the first problem (Paper 1) and in the third problem (Paper 3). In the first problem, constraints impose a minimum value for the probability of connectivity in each period, and in the third problem, an objective that maximizes the minimum value for the index over all periods is considered. Branch-and-bound and Monte Carlo tree search were developed to solve the first and third problems, respectively. It is proved that the index assumes non-increasing values in the branching step of branch-and-bound, and in the expansion and simulation steps of Monte Carlo tree search. It will be explained below how this property is important with regard to branch-and-bound.

The probability of connectivity index expresses the possibility of two individuals of a given species (or a given set of species) randomly placed in the forest falling into interconnected habitats. The index uses an indicator of a direct movement occurrence between two patches from the set of habitats, stepping stones and corridors (both mature patches smaller than a habitat and meeting a threshold area). Let Γ be this set. The indicator of a *direct movement* p_{lm} characterizes the possibility of a direct movement occurrence of an individual between two patches l and m in Γ, without passing through any intermediate patch in Γ. It may be obtained by a negative exponential function of the inter-patch distance as follows:

$$p_{lm} = e^{-\beta d_{lm}}, \tag{C.1}$$

where β is a constant greater than zero called the coefficient of dispersion (species' dependent), and d_{lm} is the edge-to-edge distance between l and m. Constant β is computed by solving equation (C.1) in order to β, where d_{lm} and p_{lm} are replaced, respectively, by a specific distance between any two pair of different patches in Γ and the expected indicator value for this distance. The closer the indicator is to 1, the smaller the inter-mature patch distance, and the more favorable the occurrence of the direct movement. The indicator is equal to 0 when two patches are completely isolated from each other.

In terms of graph theory, the mature forest can be represented by a graph, where each vertex represents a mature patch, and the distance between two mature patches is associated to the edge between their vertices. A known algorithm (*e.g.*, Dijkstra's algorithm) can be used to compute the inter-patch distance.

A path between two habitats is made up of a sequence of direct movements from one habitat to the other in which no intermediate patch in Γ is visited more than once. The *connectivity of a path* between two habitats is given by the product of the indicators of the direct movements that form the path. The largest connectivity among all paths between the habitats corresponds to the path with the greatest chance of dispersion. Let $\mathsf{g}_{hh'}$ be the largest connectivity among all paths between habitats h and h'. In case of the direct movement indicator is defined by the negative exponential function, the following proposition can be set.

Proposition C.0.1 *Let h and r be two habitats. The largest connectivity among all paths between h and h' is given by $\mathsf{g}_{hr} = e^{-\beta d}$, where d is the distance of the shortest path between h and h'.*

Proof: Consider $u_1 = h$ and $u_n = h'$. Let $R' = \{u_1, u_2, \ldots, u_n\}$ be the shortest path between h and h' in terms of distance units and d its total distance, *i.e.*, $d = d_{u_1u_2} + d_{u_2u_3} + \ldots + d_{u_{n-1}u_n}$, where $d_{u_iu_j}$ is the distance between u_i and u_j. The connectivity of R' is given by

$e^{-\beta d_{u_1u_2}} e^{-\beta d_{u_2u_3}} \ldots e^{-\beta d_{u_{n-1}u_n}} = e^{-\beta(d_{u_1u_2}+d_{u_2u_3}+\ldots+d_{u_{n-1}u_n})} = e^{-\beta d}.$

The connectivity of R' is $\mathsf{g}_{hh'} = e^{-\beta d}$, as d is the smallest distance between h and h'. ■

Let $\mathcal{H}_t$ be the set of all habitats in period t, s_h be the area of habitat h, for all $h \in \mathcal{H}_t$ and F be the total area of the forest. The probability of connectivity index for period t is given by:

$$I_t = \frac{\displaystyle\sum_{h\in\mathcal{H}_t}\sum_{h'\in\mathcal{H}_t} \mathsf{s}_h \mathsf{s}_{h'} \mathsf{g}_{hh'}}{\mathsf{F}^2}. \tag{C.2}$$

I_t ranges from 0 to 1, and increases with connectivity improvement. It is equal to 1 when all the forest is a single habitat, and is equal to zero when there are no habitats, or all habitats are completely isolated (by being too distant).

The forest of Example 1 is considered next to illustrate some of the properties of I_t that were described above.

Example 1 (continued) A habitat is a mature patch with 2 ha or more. As the whole forest is a single habitat, the index attains its maximum value (one). The distances between stands i and j, for all $i, j \in \{A, B, C, D, E\}$ are given by the matrix D

$$D = [dij] = \begin{bmatrix} 0 & 0 & 100 & 100 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 100 & 0 & 0 & 0 & 100 \\ 100 & 0 & 0 & 0 & 50 \\ 5 & 0 & 100 & 50 & 0 \end{bmatrix},$$

where d_{ij} is the distance between i and j, considering stands ordered by alphabetical order. Parameter β is equal to 0.0139 (to obtain, for a dispersal distance of 50 m, a value of 0.5 for the possibility of a direct movement). For each stand or stands harvested, the index was recalculated (I'). Table C.1 shows the new index values, as also the number of habitats and the total habitat area.

The index gradually strictly decreases with the harvest of E, C, A , B, $\{B, E\}$, $\{B, C\}$, $\{B, C, E\}$ and D. When E, C, A or D are harvested, the forest still has one habitat but

Stand(s) harvested	No. habitats	Total habitat area (ha)	I'
A	1	6.5	0.58
B	2	7	0.53
C	1	7.5	0.78
D	1	4.5	0.28
E	1	8	0.89
B and E	2	7	0.47
B and C	2	6	0.38
B, C and E	2	6	0.33

Table C.1: Number of habitats, total habitat area and connectivity index value according to the stands harvested.

progressively smaller (Property 7). When B is harvested, the whole forest is split up into two habitats. If only B is harvested, A of 2 ha and $\{C, D\}$ of 5 ha become habitats and E a stepping-stone, giving a smaller index value than when A is harvested and there is a single habitat of 6.5 ha (Property 6). When B or $\{B, E\}$ are harvested, the total area of the resulting two habitats are the same in both situations, but the value of the index is smaller in the second situation, as E cannot be used as a stepping-stone (Property 9). When $\{B, C\}$ is harvested, the forest is split up into habitats A of 2 ha and D of 4 ha and the stepping-stone E, giving a smaller index value than when $\{B, E\}$ is harvested, in which the forest is split up into habitats A of 2 ha and $\{C, D\}$ of 5 ha (Property 10). The index decreases from the harvest of $\{B, E\}$ to that of $\{B, C, E\}$, because the total habitat area is smaller in the second situation (Property 7). When D is harvested, the forest has one habitat ($\{A, B, C, E\}$) of 4.5 ha, giving a smaller index value than when $\{B, C, E\}$ is harvested, where there are two habitats A of 2 ha and D of 4 ha (Property 7). ■

In the previous example, the index value strictly decreased or maintained equal whenever a new stand was harvested. This can be settled as a property of the index as follows.

At node k of the search tree, a pair (stand, period), not selected yet, is selected to create two nodes $k + 1$ and $k + 2$ (left and right branches) corresponding to the decision of harvesting or not stand i in period t, respectively. Let I_t^k, I_t^{k+1} and I_t^{k+2} be the value of the index at nodes k, $k + 1$ and $k + 2$, respectively.

Proposition C.0.2 *Let k be a node of the search tree and $k+1$ its left child node. Harvesting a stand in period t at $k + 1$ is a sufficient condition for $I_t^k \geq I_t^{k+1}$.*

Proof: Let $\mathcal{H}_t^k$ and $\mathcal{H}_t^{k+1}$ be the sets of habitats in period t with respect to nodes k and $k+1$, respectively. Let i and t be the stand to harvest and the period in which the intervention occurs, respectively. Let $\mathsf{g}_{hh'}$ concern node k and $\mathsf{g}'_{hh'}$ be $\mathsf{g}_{hh'}$ with respect to node $k + 1$. The relation between I_t^k and I_t^{k+1} is established in the following three situations.

(i) If i is not mature in t, it does not belong to either a habitat or a stepping stone/corridor in this period. In this case, $I_t^k = I_t^{k+1}$.

(ii) If i belongs to a mature patch that is not a habitat, $\mathcal{H}_t^k = \mathcal{H}_t^{k+1}$. For each pair of habitats h and r, the distance between them in the new scenario is greater than or equal to that before the stand-period selection. Consequently, $\mathsf{g}'_{hh'}$ is smaller than or equal to $\mathsf{g}_{hh'}$, yielding $I_t^k \geq I_t^{k+1}$.

(iii) If i belongs to a habitat in node k, for example h_1, this habitat, in node $k+1$, can be replaced by (a) a mature patch that is not habitat, (b) a single habitat, or (c) several mature patches that are habitat or not.

(a) $\mathcal{H}_t^{k+1} = \mathcal{H}_t^k \setminus \{h_1\}$ and $\mathsf{g}'_{hh'} \leq \mathsf{g}_{hh'}$, $h, h' \in \mathcal{H}_t^{k+1}$.

Thus, $\displaystyle\sum_{h\in\mathcal{H}_t^{k+1}} \sum_{h'\in\mathcal{H}_t^{k+1}} \mathsf{s}_h \mathsf{s}_{h'} \mathsf{g}'_{hh'} \leq \sum_{h\in\mathcal{H}_t^{k+1}} \sum_{h'\in\mathcal{H}_t^{k+1}} \mathsf{s}_h \mathsf{s}_{h'} \mathsf{g}_{hh'} \leq \sum_{h\in\mathcal{H}_t^{k}} \sum_{r\in\mathcal{H}_t^{k}} \mathsf{s}_h \mathsf{s}_h \mathsf{g}_{hh'}$,

yielding $I_t^k \geq I_t^{k+1}$.

(b) Let a be the new habitat, that is $a = h_1 \setminus \{i\}$.

Then, $\mathsf{s}_{h_1} \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} > \mathsf{s}_a \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} \geq \mathsf{s}_a \mathsf{s}_{h'} \mathsf{g}'_{ah'}$, $h' \neq a, h_1$.

This implies that $\displaystyle\sum_{h'\in\mathcal{H}_t^k\setminus\{h_1\}} \mathsf{s}_{h_1} \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} > \sum_{h'\in\mathcal{H}_t^{k+1}\setminus\{a\}} \mathsf{s}_a \mathsf{s}_{h'} \mathsf{g}'_{ah'}$,

and thus that $\displaystyle\sum_{h'\in\mathcal{H}_t^k\setminus\{h_1\}} \mathsf{s}_{h_1} \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} + \mathsf{s}_{h_1}\mathsf{s}_{h_1} > \sum_{h'\in\mathcal{H}_t^{k+1}\setminus\{a\}} \mathsf{s}_a \mathsf{s}_{h'} \mathsf{g}'_{ah'} + \mathsf{s}_a \mathsf{s}_a$,

that means $\displaystyle\sum_{r\in\mathcal{H}_t^k} \mathsf{s}_{h_1} \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} > \sum_{h'\in\mathcal{H}_t^{k+1}} \mathsf{s}_a \mathsf{s}_{h'} \mathsf{g}'_{ah'}$.

As for each pair of habitats $h \in \mathcal{H}_t^k \cap \mathcal{H}_t^{k+1}$ and h', the distance between them in the new scenario is greater than or equal to that before the stand-period selection, then

$\displaystyle\sum_{h\in\mathcal{H}_t^{k}} \sum_{h'\in\mathcal{H}_t^{k}} \mathsf{s}_h \mathsf{s}_{h'} \mathsf{g}_{hh'} > \sum_{h\in\mathcal{H}_t^{k+1}} \sum_{h'\in\mathcal{H}_t^{k+1}} \mathsf{s}_h \mathsf{s}_{h'} \mathsf{g}'_{hh'}$, yielding $I_t^k > I_t^{k+1}$.

(c) Let L be the new set of habitats.

Then, $\displaystyle\mathsf{s}_{h_1} \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} > (\sum_{l\in L} \mathsf{s}_l) \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} = \sum_{l\in L} \mathsf{s}_l \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} \geq \sum_{l\in L} \mathsf{s}_l \mathsf{s}_{h'} \mathsf{g}'_{lh'}$,

$h' \neq h_1, h' \neq l \in L$.

This implies that $\displaystyle\sum_{h'\in\mathcal{H}_t^k\setminus\{h_1\}} \mathsf{s}_{h_1} \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} > \sum_{h'\in\mathcal{H}_t^{k+1}\setminus L} \sum_{l\in L} \mathsf{s}_l \mathsf{s}_{h'} \mathsf{g}'_{lh'}$,

and thus that $\displaystyle\sum_{r\in\mathcal{H}_t^k\setminus\{h_1\}} \mathsf{s}_{h_1} \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} + \mathsf{s}_{h_1}\mathsf{s}_{h_1} > \sum_{h'\in\mathcal{H}_t^{k+1}\setminus L} \sum_{l\in L} \mathsf{s}_l \mathsf{s}_{h'} \mathsf{g}'_{lh'} + \sum_{l,m\in L} \mathsf{s}_l \mathsf{s}_m \mathsf{g}'_{lm}$,

that means $\displaystyle\sum_{h'\in\mathcal{H}_t^k} \mathsf{s}_{h_1} \mathsf{s}_{h'} \mathsf{g}_{h_1 h'} > \sum_{h'\in\mathcal{H}_t^{k+1}} \sum_{l\in L} \mathsf{s}_l \mathsf{s}_{h'} \mathsf{g}'_{lh'}$.

As for each pair of habitats $h \in \mathcal{H}_t^k \cap \mathcal{H}_t^{k+1}$ and h', the distance between them in the new scenario is greater than or equal to that before the stand-period selection, then

$$\sum_{h \in \mathcal{H}_t^k} \sum_{h' \in \mathcal{H}_t^k} \mathsf{s}_h \mathsf{s}_{h'} \mathsf{g}_{hh'} > \sum_{h \in \mathcal{H}_t^{k+1}} \sum_{h' \in \mathcal{H}_t^{k+1}} \mathsf{s}_h \mathsf{s}_{h'} \mathsf{g}'_{hh'},\text{ yielding } I_t^k > I_t^{k+1}.$$

■

The next proposition follows immediately from the previous proposition.

Proposition C.0.3 *In the branching step of branch-and-bound, each child node has a value for the probability of connectivity index less than or equal to that of the parent node.*

Proof: Let i and t be the stand and the period selected at node k of the search tree. In right branch, $I_t^k = I_t^{k+2}$, since i is not harvested in t.

In left branch, i is harvested in t . By the previous proposition, $I_t^k \geq I_t^{k+1}$. ■

The branching property of the index plays an important role in the pruning step of branch-and-bound. Pruning is based on constraint violations and an upper bound value on the optimal solution. If a node violates the connectivity constraints, its children nodes will also violate these constraints, and thus the first node can be pruned.

Remark that this property also occurs in the expansion and simulation steps of Monte Carlo tree search. The expansion step adds two nodes according to the same strategy used in the branching step of branch-and-bound. In the simulation step, a node is randomly generated and a pair (stand, period) is selected, giving two children nodes that correspond to the decision of harvesting or not the stand in the period. However, as connectivity is not modeled by constraints in the third problem, the property is not important for the implementation of Monte Carlo tree search.

Appendix D

Implementation details

This appendix presents procedures and algorithms required by branch-and-bound and Monte Carlo tree search methods. Firstly, we describe the procedures used to check if all the required constraints are satisfied on a given tree node. The process used to determine the subregions is also briefly described. Finally, we present some well-known algorithms used by branch-and-bound and Monte Carlo tree search, such as the Dijkstra algorithm to calculate the distance between vertices in a graph, the Bron-Kerbosch algorithm to find all maximal cliques in a graph and the HSO algorithm to calculate the hypervolume measure.

D.1 Efficient constraint verification

The two branch-and-bound methods used in this thesis differ on the subset of constraints that are dealt with in each node of the tree search. Each leaf is a feasible solution of the problem considered.

Here, we consider a given node k', generated as a left or right branch of its parent k. The pair (i_k, t_k) corresponds to the stand and period fixed at node k. The values for the constraints at node k' are calculated incrementally from the corresponding values at the parent node.

Notice that if node k' was generated as a right branch — hence corresponding to a decision of not cutting (i_k, t_k) —, then only the constraint of lower bound on timber volume produced at that period needs to be checked; the values for all the other constraints are unchanged.

Follows some detail on the calculation of constraint values.

D.1.1 Volume constraints

Let $\mathcal{S}_{k'}$ be the set of all pairs $(\text{stand } i, \text{period } t)$ such that i is available to be harvested in t, sorted by descending order of the net present value of the timber available in i at period t.

The constraint of lower bound on the volume at period t_k can only be fully checked when all the decisions for that period have been taken, *i.e.*, $\mathcal{S}_{k'}$ is empty. When $\mathcal{S}_{k'}$ is not empty, we check if harvesting all stands still available for period t_k would give a volume of timber smaller that the lower bound. If so, node k' is infeasible and this node is fathomed; otherwise, no conclusion can be drawn.

When stand i_k is harvested at t_k, its volume is added to the volume of timber already harvested in t_k, previously determined in node k. If this sum is greater than the maximum limit for the volume, node k' is infeasible; otherwise, no conclusion can be drawn about the infeasibility of node k'.

D.1.2 Average ending age constraints

Mainly to prevent over-harvesting the forest, it is imposed that the average age of the forest at the end of the planning horizon should meet a treshold value, $\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$, the minimum age requirement (measured in this thesis in terms of periods).

At the root node, we compute the sum of the ages of all stands at the end of the planning horizon, weighted by their areas, given by $(T+1)\mathsf{F} + \sum_{i\in\mathcal{V}} \mathsf{age}_{i0}\mathsf{s}_i$, where age_{i0} is the initial age (in periods) of stand i (with area s_i). When i_k is harvested in t_k, the age of the harvested stand just before the intervention weighted by its area $(\mathsf{age}_{i_k0} + t_k - 1)\mathsf{s}_{i_k}$, and $(\mathsf{age}_{i_k0} + t_k)\mathsf{s}_{i_k}$ is subtracted from the sum of the ages at the parent node (it was assumed that harvestings occur at the beginning of the periods). When the updated value is less than the minimum limit $\mathsf{F}\mathsf{Age}^{\mathsf{min}}_{\mathsf{end}}$, where F is the forest area, node k' is infeasible; otherwise, no conclusion can be drawn about the infeasibility of node k'.

D.1.3 Maximum clearcut size constraints

When stand i_k is harvested at period t_k, either its area is added to an existing adjacent clearcut, or a new clearcut is formed with that stand. In both cases, it is necessary to check if the updated area is greater than $\mathsf{A}^{\mathsf{max}}$. If this happens, node k' is infeasible; otherwise, no conclusion can be drawn about the infeasibility of node k'.

D.1.4 Total habitat area constraints

When harvesting a stand i_k at period t_k, the number of periods where i_k is not mature is given by *tmax* $= \min\{t_k + (\mathsf{Age}^{\mathsf{min}}_{\mathsf{old}} - 1), T\}$. At a children node k', total habitat area constraints must be checked from period $u = t_k$ to *tmin*. The following notation is used. We define $\mathcal{H}^k_t$ as the set of existing habitats in period t for node k, and tarea^k_t the total area of habitats in $\mathcal{H}^k_t$. In period u, two situations may occur:

1. If stand i_k belongs to some habitat $h \in \mathcal{H}^k_u$ and harvesting i_k leads to a smaller patch

h_1 (Figure D.1 (a)), then:

- if h_1 meets the minimum requirement for a habitat:
  - $\mathrm{tarea}_u^{k'}$ is equal to tarea_u^k minus the area of i_k;
  - $\mathcal{H}_u^{k'}$ is obtained replacing h into h_1 in $\mathcal{H}_u^k$;
- if h_1 does not meet the minimum requirement for a habitat:
  - $\mathrm{tarea}_u^{k'}$ is equal to tarea_u^k minus the area of h;
  - $\mathcal{H}_u^{k'}$ is obtained removing h from $\mathcal{H}_u^k$;

2. If stand i_k belongs to habitat $h \in \mathcal{H}_u^k$ and harvesting i_k splits up h into several patches h_m (Figure D.1 (b)), then:
   - $\mathrm{tarea}_u^{k'}$ is initialized by subtracting the the area of i_k from tarea_u^k;
   - $\mathcal{H}_u^{k'}$ is initialized removing h from $\mathcal{H}_u^k$;
   - for each h_m that meets the minimum requirement for a habitat, h_m is added to $\mathcal{H}_u^{k'}$;
   - for each h_m that is non-habitat, the area of h_m is subtracted from $\mathrm{tarea}_u^{k'}$ (and h_m is not inserted into $\mathcal{H}_u^{k'}$).

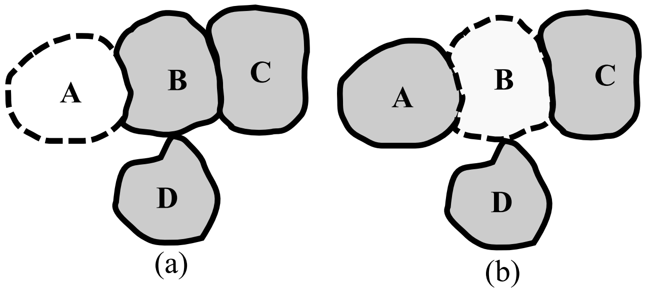

Figure D.1: Mature forest with four stands, where patch $\{A, B, C, D\}$ is habitat. After harvesting stands A or B (left and right, respectively), each of the dotted areas is subtracted from the habitat area.

If $\mathrm{tarea}_u^{k'}$ is less than the minimum total habitat area $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}$ for some period u, node k' is infeasible; otherwise, no conclusion can be drawn about the infeasibility of node k'.

D.1.5 Total habitat area and total core area constraints (papers 2 and 3)

It is assumed that edge effects occurs only after harvesting and during the period of intervention, *i.e.*, in period t_k, for which total habitat and core area constraints need to be checked.

Periods where stand i_k is not mature and edge effects are not considered are $u = t_k + 1$ to *tmax*; if i_k is mature before the intervention, only total habitat area constraints needed to be checked for these periods (as described in D.1.4). Otherwise, nor do these constraints need to be verified.

Concerning period t_k, let carea_t^k be the total core area of habitats in $\mathcal{H}_t^k$. Three situations may occur when i_k is harvested:

1. If stand i_k belongs to habitat $h \in \mathcal{H}_{t_k}^k$ and harvesting i_k leads to a smaller patch h_1 (Figure D.2 (a)), then:

   - if h_1 meets the requirement for a habitat:
     - the new core area of h_1 is the core area of h minus R, where R is the core area of h that was inside i_k plus the newly created edge in h_1 caused by harvesting i_k;
     - $\mathrm{tarea}_{t_k}^{k'}$ is equal to $\mathrm{tarea}_{t_k}^{k}$ minus the area of i_k;
     - $\mathrm{carea}_{t_k}^{k'}$ is equal to $\mathrm{carea}_{t_k}^{k}$ minus R;
     - $\mathcal{H}_{t_k}^{k'}$ is obtained replacing h into h_1 in $\mathcal{H}_{t_k}^{k}$;
   - if h_1 does not meet the minimum area requirement for a habitat, then
     - $\mathrm{tarea}_{t_k}^{k'}$ is equal to $\mathrm{tarea}_{t_k}^{k}$ minus the area of h;
     - $\mathrm{carea}_{t_k}^{k'}$ is equal to $\mathrm{carea}_{t_k}^{k}$ minus the core area of h;
     - $\mathcal{H}_{t_k}^{k'}$ is obtained removing h from $\mathcal{H}_{t_k}^{k}$.

2. If stand i_k belongs to habitat $h \in \mathcal{H}_{t_k}^k$ and harvesting i_k splits up h into several patches h_m (Figure D.2 (b)), then:

   - $\mathrm{tarea}_{t_k}^{k'}$ is initialized by subtracting the area of i_k from $\mathrm{tarea}_{t_k}^{k}$;
   - $\mathrm{carea}_{t_k}^{k'}$ is initialized by subtracting the core area that was inside of i_k from $\mathrm{carea}_{t_k}^{k}$;
   - $\mathcal{H}_{t_k}^{k'}$ is initialized removing h from $\mathcal{H}_{t_k}^{k}$;
   - for each h_m that meets the minimum requirement for a habitat, the newly created edge caused by harvesting i_k is subtracted from $\mathrm{carea}_{t_k}^{k'}$ and h_m is added to $\mathcal{H}_{t_k}^{k'}$;
   - for each h_m that is non-habitat, the area of h_m is subtracted from $\mathrm{tarea}_{t_k}^{k'}$, the core area that is inside of h_m is subtracted from $\mathrm{carea}_{t_k}^{k'}$ (and h_m is not inserted into $\mathcal{H}_{t_k}^{k'}$).

3. If stand i_k (belonging or not to habitat $h \in \mathcal{H}_{t_k}^k$) causes edge effects on other habitats $h_s \in \mathcal{H}_{t_k}^k$ (Figure D.2 (c)):

   - the core area of each h_s is updated by subtracting the newly created edge caused by harvesting i_k;
   - for each h_s that remains habitat, the newly created edge is subtracted from $\mathrm{carea}_{t_k}^{k'}$ (and $\mathcal{H}_{t_k}^{k'}$ are not updated);

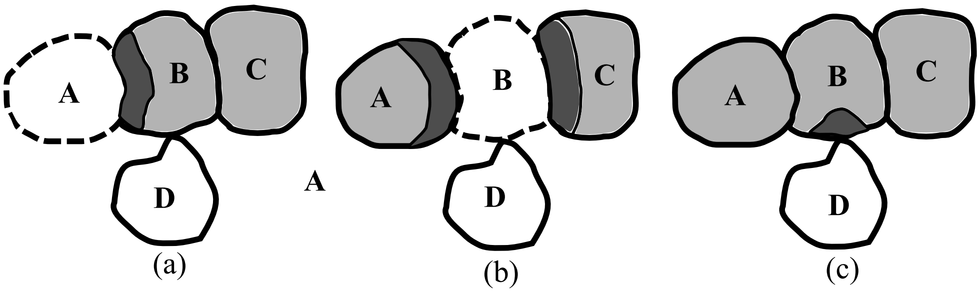

Figure D.2: Mature forest with four stands, where patch $\{A, B, C\}$ is habitat. After harvesting stands A, B or D (left, middle and right, respectively), the dark shaded areas and the dotted areas are subtracted from the habitat core area, and the dotted areas are subtracted from the habitat area.

- for each h_s that becomes non-habitat, the area and the core area of h_s are subtracted from $\mathrm{tarea}_{t_k}^{k'}$ and $\mathrm{carea}_{t_k}^{k'}$, respectively, and h_s is removed from $\mathcal{H}_{t_k}^{k'}$.

If $\mathrm{tarea}_{t_k}^{k'}$ or $\mathrm{carea}_{t_k}^{k'}$ are less than the respective lower bounds $\mathsf{H}_{\mathsf{tot}}^{\mathsf{min}}$ and $\mathsf{C}_{\mathsf{tot}}^{\mathsf{min}}$, node k' is infeasible; otherwise, no conclusion can be drawn about the infeasibility of node k'.

D.1.6 Connectivity constraints (paper 1)

From period $u = t_k$ to *tmax*, where *tmax* $= \min\{t_k + (\mathsf{Age}_{\mathsf{old}}^{\mathsf{min}} - 1), T\}$ (periods where i_k is not mature), the connectivity index I_u must be recalculated. If its value is less than the lower bound I for some period u, then node k is infeasible; otherwise, no conclusion can be drawn about the infeasibility of node k'.

D.2 Determining subregions (papers 2 and 3)

For each of the instances considered in this thesis, the forest is partitioned into subregions with the geographic information system ArcGis 9.2.

A surrounding impact zone of a given width for each stand is created using the tool *Buffer,* available in *ArcToolbox* $\backslash$ *Analysis Tools* $\backslash$ *Proximity*. Then, subregions are created using *ArcToolbox* $\backslash$ *Analysis Tools* $\backslash$ *Overlay*. For each subregion, ArcGis outputs its area and the set of stands defining it. Centroids were computed using the tools *Feature to Point* and *Add XY Coordinates* from *ArcToolbox* $\backslash$ *Data Management Tools* $\backslash$ *Features*, and used to distinguish subregions with the same defining set of stands and the same area.

D.3 Dijkstra's algorithm (papers 1 and 3)

Dijkstra's algorithm is applied to determine the shortest path from a habitat to other, where intermediate mature patches can be used in the path. Each mature patch is represented by a vertex of a graph and the distance between two mature patches is the weight associated to the respective edge. For computing the distance between two patches, we consider the distance between two stands (represented as polygons) as the minimum Euclidean distance between their vertices. The edge-to-edge distance between two mature patches is approximated by the minimum distance between their stands.

D.3.1 General description

Dijkstra's algorithm finds the shortest path from a vertex to other (or the shortest paths to all other vertices) in a graph with non-negatives weights. It was developed by Esger Dijkstra in 1956 [Dijkstra, 1959]. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph with non-negatives weights. Let v_b denote the initial vertex, $d(v)$ the distance from the vertex v_b to vertex v and $w(v, v')$ the weight of edge $\{v, v'\}$.

Algorithm 3 gives an overview of the steps of the algorithm. The objective is to determine the shortest path from v_b to v_e (or from v_b to all vertices). The distance $d(v_b)$ is initialized to 0 and the distances of all other vertices are initialized to ∞. This algorithm searches vertices in a non-decreasing order of the distance of the vertices. So, the vertex i with the minimum distance from the set of unvisited vertices is reached and it is marked as visited. For all j marked as unvisited, the edge $\{i, j\}$ is checked and the distance $d(j)$ is improved if $d(i) + w(i, j) < d(j)$. The process is repeated until vertex t is marked as visited.

Notice that if all vertices are marked as visited, the algorithm ends with the shortest paths from s to all other vertices.

Algorithm 3: Main steps of Dijkstra algorithm

Step 1 **Initialization:** set $d(v_b) = 0$ and $d(i) = \infty$, for $i \neq v_b$

Step 2 **Termination:**
if v_e *is marked as visited* **then** terminate

Step 3 **Vertex selection:**
select the vertex i with the minimum value of d from the set of all unvisited vertices
marked i as visited

Step 4 Updating:
for j *such that exists the edge* (i, j) *and* j *is marked as unvisited* **do**
    $d(j) := min\{d(j), d(i) + w(i, j)\}$
go to Step 2

Let n be the number of vertices and m_e the number of edges of the input graph $\mathcal{G}$. When the graph is represented by an adjacency list and it is used a heap (priority queue) for the unvisited vertices, the running time is $\mathcal{O}(m_e\ log\ n)$.

D.4 Bron-Kerbosch algorithm

Bron-Kerbosch algorithm is used to find all maximal cliques in a graph, where each vertex corresponds to a stand of the forest and the endpoints of each edge correspond to two adjacent stands according to the definition of adjacency.

D.4.1 General description

The algorithm of Bron and Kerbosch [Bron and Kerbosch, 1973] is a widely used algorithm to find all maximal cliques in a graph.

Algorithm 4 performs the pseudocode of a recursive Bron-Kerbosch algorithm. It provides three disjoint sets of vertices R_1, R_2, and R_3 as arguments, where R_1 is a (possibly non-maximal) clique, and R_2 and R_3 are disjoint sets whose union consists of those vertices that form cliques when added to R_1. The vertices in R_2 will be considered to be added to clique R_1, while those in R_3 must be excluded from the clique; thus, within the recursive call, the algorithm lists all cliques in $R_1 \cup R_2$ that are maximal within the subgraph induced by $R_1 \cup R_2 \cup R_3$. Let $N(v)$ be the neighbour set of vertex v, *i.e.*, the set of adjacent stands of v . The algorithm is called with P equal to the set of all vertices in the graph and with R_1 and R_3 empty. The algorithm chooses a pivot vertex v', chosen from $R_2 \cup R_3$ and it makes a recursive call in which v in $R_2 \setminus N(v')$ is added to R_1 and in which R_2 and R_3 are restricted to the neighbour set $N(v)$. When the recursive call returns, v is moved from R_2 to R_3 to eliminate redundant work by further calls to the algorithm. When the recursion reaches a level at $R_2 \cup R_3 = \emptyset$, R_1 is a maximal clique and it is reported.

Algorithm 4: Bron-Kerbosch algorithm

```
Bron-Kerbosch(R1, R2, R3)
if R2 ∪ R3 = ∅ then report R1 as a maximal clique
choose a pivot vertex v' in R2 ∪ R3
for each vertex v in R2 \ N(v') do
    BronKerbosch(R1 ∪ v, R2 ∩ N(v), R3 ∩ N(v))
    remove v from R2
    add v to R3
```

Tomita et al. [2006] show that choosing the pivot v' from $R_2 \cup R_3$ as the vertex with the highest number of neighbours in R_2 guarantees that the worst-case running of BronKerbosch algorithm is $\mathcal{O}(3^{n/3})$, where n is the number of vertices.

D.5 HSO hypervolume algorithm (paper 3)

The Hypervolume by Slicing Objectives (HSO) calculates the hypervolume of each Pareto front reached by the multi-objective Monte Carlo tree search.

D.5.1 General description

HSO algorithm [While et al., 2006] processes a Pareto front by processing one objective at a time, slicing along the chosen objective. Consider p objectives (or dimensions), an input set of n points and, for example, the input points sorted in order to objective p. Each point defines the bottom of a slice, and the next point defines the top of that slice. The hipervolume of each slice is calculated recursively by sweeping along the $p-1$ objective, calculating the hypervolume of the resulting $(p-1)$-dimensional slices until a base of dimension 2, multiplied by the depth of the slice (distance beteween the two points). The sum of the hypervolume of all p-dimensional slices gives the exact value of the hypervolume. In particular, for two objectives ($p = 2$), the hypervolume is calculated by sorting the points in order to one of the objectives and adding the areas of the rectangles defined by each point and the next one. Algorithm 5 provides a recursive algorithm to calculate the hypervolume HV, where list S is sorted in order to objective p.

Algorithm 5: HSO hypervolume algorithm

HSO(S, p**)**
if $p = 2$ **then** return HV2D(S)
create empty list S'
for *each point* v *in* S **do**
    d= distance from v to the next point in S or the reference point if v is the final point
    insert ($S', v, p-1$)
    $HV = HV + d$ HSO($S', p-1$)

insert(S, v, i)
  insert v in S, maintaining S sorted in order to objective i
  delete all points dominated by v
HV2D(S)
  return the hypervolume HV of 2 objectives of S

While et al. [2006] proved that HSO is exponential in the number of objectives. The hypervolume can be calculated in linear time when $p = 2$.